
Review Problems

17-118 A leak develops in an automobile tire as a result of an accident. The initial mass flow rate of air through the leak is to be determined.

Assumptions 1 Air is an ideal gas with constant specific heats. **2** Flow of air through the hole is isentropic.

Properties The gas constant of air is $R = 0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$. The specific heat ratio of air at room temperature is $k = 1.4$ (Table A-2a).

Analysis The absolute pressure in the tire is

$$P = P_{\text{gage}} + P_{\text{atm}} = 220 + 94 = 314 \text{ kPa}$$

The critical pressure is, from Table 17-2,

$$P^* = 0.5283P_0 = (0.5283)(314 \text{ kPa}) = 166 \text{ kPa} > 94 \text{ kPa}$$

Therefore, the flow is choked, and the velocity at the exit of the hole is the sonic speed. Then the flow properties at the exit becomes

$$\rho_0 = \frac{P_0}{RT_0} = \frac{314 \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298 \text{ K})} = 3.671 \text{ kg/m}^3$$

$$\rho^* = \rho_0 \left(\frac{2}{k+1} \right)^{1/(k-1)} = (3.671 \text{ kg/m}^3) \left(\frac{2}{1.4+1} \right)^{1/(1.4-1)} = 2.327 \text{ kg/m}^3$$

$$T^* = \frac{2}{k+1} T_0 = \frac{2}{1.4+1} (298 \text{ K}) = 248.3 \text{ K}$$

$$V = c = \sqrt{kRT^*} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right) (248.3 \text{ K})} = 315.9 \text{ m/s}$$

Then the initial mass flow rate through the hole becomes

$$\dot{m} = \rho AV = (2.327 \text{ kg/m}^3) [\pi(0.004 \text{ m})^2/4] (315.9 \text{ m/s}) = 0.00924 \text{ kg/s} = \mathbf{0.554 \text{ kg/min}}$$

Discussion The mass flow rate will decrease with time as the pressure inside the tire drops.

17-119 The thrust developed by the engine of a Boeing 777 is about 380 kN. The mass flow rate of air through the nozzle is to be determined.

Assumptions **1** Air is an ideal gas with constant specific properties. **2** Flow of combustion gases through the nozzle is isentropic. **3** Choked flow conditions exist at the nozzle exit. **4** The velocity of gases at the nozzle inlet is negligible.

Properties The gas constant of air is $R = 0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ (Table A-1), and it can also be used for combustion gases. The specific heat ratio of combustion gases is $k = 1.33$ (Table 17-2).

Analysis The velocity at the nozzle exit is the sonic velocity, which is determined to be

$$V = c = \sqrt{kRT} = \sqrt{(1.33)(0.287 \text{ kJ/kg}\cdot\text{K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)(265 \text{ K})} = 318.0 \text{ m/s}$$

Noting that thrust F is related to velocity by $F = \dot{m}V$, the mass flow rate of combustion gases is determined to be

$$\dot{m} = \frac{F}{V} = \frac{380,000 \text{ N}}{318.0 \text{ m/s}} \left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right) = \mathbf{1194.8 \text{ kg/s}}$$

Discussion The combustion gases are mostly nitrogen (due to the 78% of N_2 in air), and thus they can be treated as air with a good degree of approximation.

17-120 A stationary temperature probe is inserted into an air duct reads 85°C . The actual temperature of air is to be determined.

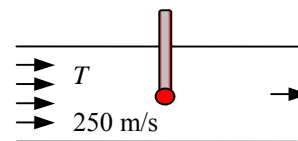
Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** The stagnation process is isentropic.

Properties The specific heat of air at room temperature is $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$ (Table A-2a).

Analysis The air that strikes the probe will be brought to a complete stop, and thus it will undergo a stagnation process. The thermometer will sense the temperature of this stagnated air, which is the stagnation temperature. The actual air temperature is determined from

$$T = T_0 - \frac{V^2}{2c_p} = 85^\circ\text{C} - \frac{(250 \text{ m/s})^2}{2 \times 1.005 \text{ kJ/kg}\cdot\text{K}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = \mathbf{53.9^\circ\text{C}}$$

Discussion Temperature rise due to stagnation is very significant in high-speed flows, and should always be considered when compressibility effects are not negligible.

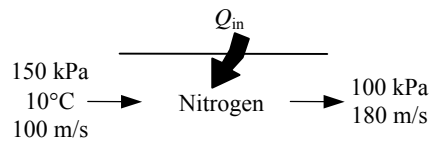


17-121 Nitrogen flows through a heat exchanger. The stagnation pressure and temperature of the nitrogen at the inlet and the exit states are to be determined.

Assumptions 1 Nitrogen is an ideal gas with constant specific properties. 2 Flow of nitrogen through the heat exchanger is isentropic.

Properties The properties of nitrogen are $c_p = 1.039$ kJ/kg·K and $k = 1.4$ (Table A-2a).

Analysis The stagnation temperature and pressure of nitrogen at the inlet and the exit states are determined from



$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 10^\circ\text{C} + \frac{(100 \text{ m/s})^2}{2 \times 1.039 \text{ kJ/kg} \cdot ^\circ\text{C}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = \mathbf{14.8^\circ\text{C}}$$

$$P_{01} = P_1 \left(\frac{T_{01}}{T_1} \right)^{k/(k-1)} = (150 \text{ kPa}) \left(\frac{288.0 \text{ K}}{283.2 \text{ K}} \right)^{1.4/(1.4-1)} = \mathbf{159.1 \text{ kPa}}$$

From the energy balance relation $E_{in} - E_{out} = \Delta E_{system}$ with $w = 0$

$$q_{in} = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} + \Delta pe \approx 0$$

$$125 \text{ kJ/kg} = (1.039 \text{ kJ/kg} \cdot ^\circ\text{C})(T_2 - 10^\circ\text{C}) + \frac{(180 \text{ m/s})^2 - (100 \text{ m/s})^2}{2} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)$$

$$T_2 = 119.5^\circ\text{C}$$

and,

$$T_{02} = T_2 + \frac{V_2^2}{2c_p} = 119.5^\circ\text{C} + \frac{(180 \text{ m/s})^2}{2 \times 1.039 \text{ kJ/kg} \cdot ^\circ\text{C}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = \mathbf{135.1^\circ\text{C}}$$

$$P_{02} = P_2 \left(\frac{T_{02}}{T_2} \right)^{k/(k-1)} = (100 \text{ kPa}) \left(\frac{408.3 \text{ K}}{392.7 \text{ K}} \right)^{1.4/(1.4-1)} = \mathbf{114.6 \text{ kPa}}$$

Discussion Note that the stagnation temperature and pressure can be very different than their thermodynamic counterparts when dealing with compressible flow.

17-122 An expression for the speed of sound based on van der Waals equation of state is to be derived. Using this relation, the speed of sound in carbon dioxide is to be determined and compared to that obtained by ideal gas behavior.

Properties The properties of CO₂ are $R = 0.1889 \text{ kJ/kg}\cdot\text{K}$ and $k = 1.279$ at $T = 50^\circ\text{C} = 323.2 \text{ K}$ (Table A-2b).

Analysis Van der Waals equation of state can be expressed as $P = \frac{RT}{v-b} - \frac{a}{v^2}$.

$$\text{Differentiating, } \left(\frac{\partial P}{\partial v}\right)_T = \frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

Noting that $\rho = 1/v \longrightarrow d\rho = -dv/v^2$, the speed of sound relation becomes

$$\begin{aligned} c^2 &= k \left(\frac{\partial P}{\partial \rho}\right)_T = v^2 k \left(\frac{\partial P}{\partial v}\right)_T \\ \text{Substituting,} \quad c^2 &= \frac{v^2 kRT}{(v-b)^2} - \frac{2ak}{v} \end{aligned}$$

Using the molar mass of CO₂ ($M = 44 \text{ kg/kmol}$), the constant a and b can be expressed per unit mass as

$$a = 0.1882 \text{ kPa}\cdot\text{m}^6/\text{kg}^2 \quad \text{and} \quad b = 9.70 \times 10^{-4} \text{ m}^3/\text{kg}$$

The specific volume of CO₂ is determined to be

$$200 \text{ kPa} = \frac{(0.1889 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323.2 \text{ K})}{v - 0.000970 \text{ m}^3/\text{kg}} - \frac{2 \times 0.1882 \text{ kPa}\cdot\text{m}^6/\text{kg}^2}{v^2} \rightarrow v = 0.300 \text{ m}^3/\text{kg}$$

Substituting,

$$c = \left(\frac{\left(\frac{(0.300 \text{ m}^3/\text{kg})^2 (1.279)(0.1889 \text{ kJ/kg}\cdot\text{K})(323.2 \text{ K})}{(0.300 - 0.000970 \text{ m}^3/\text{kg})^2} \right) \frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}}{\frac{2(0.1882 \text{ kPa}\cdot\text{m}^6/\text{kg}^3)(1.279)}{(0.300 \text{ m}^3/\text{kg})^2} \frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kPa}\cdot\text{m}^3/\text{kg}}} \right)^{1/2} = \mathbf{271 \text{ m/s}}$$

If we treat CO₂ as an ideal gas, the speed of sound becomes

$$c = \sqrt{kRT} = \sqrt{(1.279)(0.1889 \text{ kJ/kg}\cdot\text{K})(323.2 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = \mathbf{279 \text{ m/s}}$$

Discussion Note that the ideal gas relation is the simplest equation of state, and it is very accurate for most gases encountered in practice. At high pressures and/or low temperatures, however, the gases deviate from ideal gas behavior, and it becomes necessary to use more complicated equations of state.

17-123 The equivalent relation for the speed of sound is to be verified using thermodynamic relations.

Analysis The two relations are $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s$ and $c^2 = k \left(\frac{\partial P}{\partial \rho}\right)_T$

From $r = 1/\nu \longrightarrow dr = -d\nu/\nu^2$. Thus,

$$c^2 = \left(\frac{\partial P}{\partial r}\right)_s = -\nu^2 \left(\frac{\partial P}{\partial \nu}\right)_s = -\nu^2 \left(\frac{\partial P}{\partial T} \frac{\partial T}{\partial \nu}\right)_s = -\nu^2 \left(\frac{\partial P}{\partial T}\right)_s \left(\frac{\partial T}{\partial \nu}\right)_s$$

From the cyclic rule,

$$(P, T, s): \left(\frac{\partial P}{\partial T}\right)_s \left(\frac{\partial T}{\partial s}\right)_P \left(\frac{\partial s}{\partial P}\right)_T = -1 \longrightarrow \left(\frac{\partial P}{\partial T}\right)_s = -\left(\frac{\partial s}{\partial T}\right)_P \left(\frac{\partial P}{\partial s}\right)_T$$

$$(T, \nu, s): \left(\frac{\partial T}{\partial \nu}\right)_s \left(\frac{\partial \nu}{\partial s}\right)_T \left(\frac{\partial s}{\partial T}\right)_\nu = -1 \longrightarrow \left(\frac{\partial T}{\partial \nu}\right)_s = -\left(\frac{\partial s}{\partial \nu}\right)_T \left(\frac{\partial T}{\partial s}\right)_\nu$$

Substituting,

$$c^2 = -\nu^2 \left(\frac{\partial s}{\partial T}\right)_P \left(\frac{\partial P}{\partial s}\right)_T \left(\frac{\partial s}{\partial \nu}\right)_T \left(\frac{\partial T}{\partial s}\right)_\nu = -\nu^2 \left(\frac{\partial s}{\partial T}\right)_P \left(\frac{\partial T}{\partial s}\right)_\nu \left(\frac{\partial P}{\partial s}\right)_T$$

Recall that

$$\frac{c_p}{T} = \left(\frac{\partial s}{\partial T}\right)_P \quad \text{and} \quad \frac{c_\nu}{T} = \left(\frac{\partial s}{\partial T}\right)_\nu$$

Substituting,

$$c^2 = -\nu^2 \left(\frac{c_p}{T}\right) \left(\frac{T}{c_\nu}\right) \left(\frac{\partial P}{\partial \nu}\right)_T = -\nu^2 k \left(\frac{\partial P}{\partial \nu}\right)_T$$

Replacing $-d\nu/\nu^2$ by $d\rho$,

$$c^2 = k \left(\frac{\partial P}{\partial \rho}\right)_T$$

Discussion Note that the differential thermodynamic property relations are very useful in the derivation of other property relations in differential form.

17-124 For ideal gases undergoing isentropic flows, expressions for P/P^* , T/T^* , and ρ/ρ^* as functions of k and Ma are to be obtained.

Analysis Equations 17-18 and 17-21 are given to be $\frac{T_0}{T} = \frac{2 + (k-1)\text{Ma}^2}{2}$ and $\frac{T^*}{T_0} = \frac{2}{k+1}$

Multiplying the two, $\left(\frac{T_0}{T} \frac{T^*}{T_0}\right) = \left(\frac{2 + (k-1)\text{Ma}^2}{2}\right) \left(\frac{2}{k+1}\right)$

Simplifying and inverting, $\frac{T}{T^*} = \frac{k+1}{2 + (k-1)\text{Ma}^2}$ (1)

From $\frac{P}{P^*} = \left(\frac{T}{T^*}\right)^{k/(k-1)} \longrightarrow \frac{P}{P^*} = \left(\frac{k+1}{2 + (k-1)\text{Ma}^2}\right)^{k/(k-1)}$ (2)

From $\frac{\rho}{\rho^*} = \left(\frac{P}{P^*}\right)^{k/(k-1)} \longrightarrow \frac{\rho}{\rho^*} = \left(\frac{k+1}{2 + (k-1)\text{Ma}^2}\right)^{k/(k-1)}$ (3)

Discussion Note that some very useful relations can be obtained by very simple manipulations.

17-125 It is to be verified that for the steady flow of ideal gases $dT_0/T = dA/A + (1-\text{Ma}^2) dV/V$. The effect of heating and area changes on the velocity of an ideal gas in steady flow for subsonic flow and supersonic flow are to be explained.

Analysis We start with the relation $\frac{V^2}{2} = c_p(T_0 - T)$, (1)

Differentiating, $V dV = c_p(dT_0 - dT)$ (2)

We also have $\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$ (3)

and $\frac{dP}{\rho} + V dV = 0$ (4)

Differentiating the ideal gas relation $P = \rho RT$, $\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} = 0$ (5)

From the speed of sound relation, $c^2 = kRT = (k-1)c_p T = kP/\rho$ (6)

Combining Eqs. (3) and (5), $\frac{dP}{P} - \frac{dT}{T} + \frac{dA}{A} + \frac{dV}{V} = 0$ (7)

Combining Eqs. (4) and (6), $\frac{dP}{\rho} = \frac{dP}{kP/c^2} = -V dV$

or, $\frac{dP}{P} = -\frac{k}{c^2} V dV = -k \frac{V^2}{c^2} \frac{dV}{V} = -k \text{Ma}^2 \frac{dV}{V}$ (8)

Combining Eqs. (2) and (6), $dT = dT_0 - V \frac{dV}{c_p}$

or, $\frac{dT}{T} = \frac{dT_0}{T} - \frac{V^2}{c_p T} \frac{dV}{V} = \frac{dT}{T} = \frac{dT_0}{T} - \frac{V^2}{c^2/(k-1)} \frac{dV}{V} = \frac{dT_0}{T} - (k-1) \text{Ma}^2 \frac{dV}{V}$ (9)

Combining Eqs. (7), (8), and (9), $-(k-1) \text{Ma}^2 \frac{dV}{V} - \frac{dT_0}{T} + (k-1) \text{Ma}^2 \frac{dV}{V} + \frac{dA}{A} + \frac{dV}{V} = 0$

or, $\frac{dT_0}{T} = \frac{dA}{A} + [-k \text{Ma}^2 + (k-1) \text{Ma}^2 + 1] \frac{dV}{V}$

Thus, $\frac{dT_0}{T} = \frac{dA}{A} + (1 - \text{Ma}^2) \frac{dV}{V}$ (10)

Differentiating the steady-flow energy equation $q = h_{02} - h_{01} = c_p(T_{02} - T_{01})$

$$\delta q = c_p dT_0 \quad (11)$$

Eq. (11) relates the stagnation temperature change dT_0 to the net heat transferred to the fluid. Eq. (10) relates the velocity changes to area changes dA , and the stagnation temperature change dT_0 or the heat transferred.

(a) When $\text{Ma} < 1$ (subsonic flow), the fluid will accelerate if the duct converges ($dA < 0$) or the fluid is heated ($dT_0 > 0$ or $\delta q > 0$). The fluid will decelerate if the duct diverges ($dA > 0$) or the fluid is cooled ($dT_0 < 0$ or $\delta q < 0$).

(b) When $\text{Ma} > 1$ (supersonic flow), the fluid will accelerate if the duct diverges ($dA > 0$) or the fluid is cooled ($dT_0 < 0$ or $\delta q < 0$). The fluid will decelerate if the duct converges ($dA < 0$) or the fluid is heated ($dT_0 > 0$ or $\delta q > 0$).

17-126 A pitot tube measures the difference between the static and stagnation pressures for a subsonic airplane. The speed of the airplane and the flight Mach number are to be determined.

Assumptions **1** Air is an ideal gas with constant specific heat ratio. **2** The stagnation process is isentropic.

Properties The properties of air are $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ and $k = 1.4$ (Table A-2a).

Analysis The stagnation pressure of air at the specified conditions is

$$P_0 = P + \Delta P = 70.109 + 35 = 105.109 \text{ kPa}$$

Then,

$$\frac{P_0}{P} = \left(1 + \frac{(k-1)\text{Ma}^2}{2}\right)^{k/k-1} \longrightarrow \frac{105.109}{70.109} = \left(1 + \frac{(1.4-1)\text{Ma}^2}{2}\right)^{1.4/0.4}$$

It yields $\text{Ma} = \mathbf{0.783}$

The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(268.65 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 328.5 \text{ m/s}$$

Thus,

$$V = \text{Ma} \times c = (0.783)(328.5 \text{ m/s}) = \mathbf{257.3 \text{ m/s}}$$

Discussion Note that the flow velocity can be measured in a simple and accurate way by simply measuring pressure.

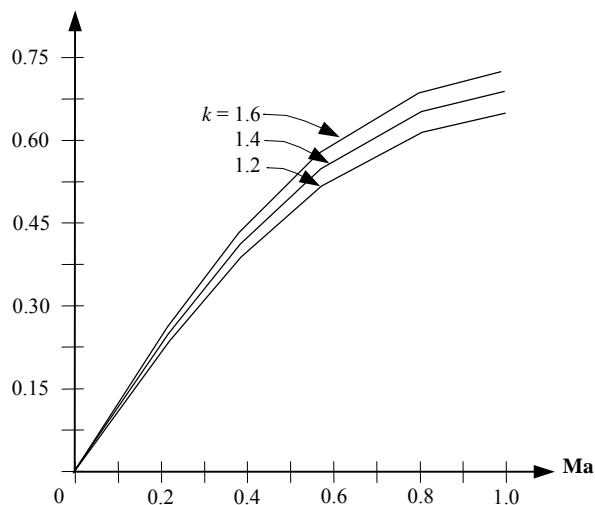
17-127 The mass flow parameter $\dot{m}\sqrt{RT_0} / (AP_0)$ versus the Mach number for $k = 1.2, 1.4,$ and 1.6 in the range of $0 \leq \text{Ma} \leq 1$ is to be plotted.

Analysis The mass flow rate parameter $(\dot{m}\sqrt{RT_0}) / P_0 A$ can be expressed as

$$\frac{\dot{m}\sqrt{RT_0}}{P_0 A} = \text{Ma} \sqrt{k} \left(\frac{2}{2 + (k-1)\text{Ma}^2} \right)^{(k+1)/2(k-1)}$$

Thus,

Ma	$k = 1.2$	$k = 1.4$	$k = 1.6$
0.0	0	0	0
0.1	0.1089	0.1176	0.1257
0.2	0.2143	0.2311	0.2465
0.3	0.3128	0.3365	0.3582
0.4	0.4015	0.4306	0.4571
0.5	0.4782	0.5111	0.5407
0.6	0.5411	0.5763	0.6077
0.7	0.5894	0.6257	0.6578
0.8	0.6230	0.6595	0.6916
0.9	0.6424	0.6787	0.7106
1.0	0.6485	0.6847	0.7164



Discussion Note that the mass flow rate increases with increasing Mach number and specific heat ratio. It levels off at $\text{Ma} = 1$, and remains constant (choked flow).

17-128 Helium gas is accelerated in a nozzle. The pressure and temperature of helium at the location where $Ma = 1$ and the ratio of the flow area at this location to the inlet flow area are to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

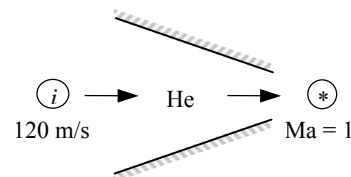
Properties The properties of helium are $R = 2.0769 \text{ kJ/kg}\cdot\text{K}$, $c_p = 5.1926 \text{ kJ/kg}\cdot\text{K}$, and $k = 1.667$ (Table A-2a).

Analysis The properties of the fluid at the location where $Ma = 1$ are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i + \frac{V_i^2}{2c_p} = 500 \text{ K} + \frac{(120 \text{ m/s})^2}{2 \times 5.1926 \text{ kJ/kg}\cdot\text{K}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 501.4 \text{ K}$$

and

$$P_0 = P_i \left(\frac{T_0}{T_i} \right)^{k/(k-1)} = (0.8 \text{ MPa}) \left(\frac{501.4 \text{ K}}{500 \text{ K}} \right)^{1.667/(1.667-1)} = 0.806 \text{ MPa}$$



The Mach number at the nozzle exit is given to be $Ma = 1$. Therefore, the properties at the nozzle exit are the *critical properties* determined from

$$T^* = T_0 \left(\frac{2}{k+1} \right) = (501.4 \text{ K}) \left(\frac{2}{1.667+1} \right) = 376 \text{ K}$$

$$P^* = P_0 \left(\frac{2}{k+1} \right)^{k/(k-1)} = (0.806 \text{ MPa}) \left(\frac{2}{1.667+1} \right)^{1.667/(1.667-1)} = 0.393 \text{ MPa}$$

The speed of sound and the Mach number at the nozzle inlet are

$$c_i = \sqrt{kRT_i} = \sqrt{(1.667)(2.0769 \text{ kJ/kg}\cdot\text{K})(500 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 1316 \text{ m/s}$$

$$Ma_i = \frac{V_i}{c_i} = \frac{120 \text{ m/s}}{1316 \text{ m/s}} = 0.0912$$

The ratio of the entrance-to-throat area is

$$\begin{aligned} \frac{A_i}{A^*} &= \frac{1}{Ma_i} \left[\left(\frac{2}{k+1} \right) \left(1 + \frac{k-1}{2} Ma_i^2 \right) \right]^{(k+1)/[2(k-1)]} \\ &= \frac{1}{0.0912} \left[\left(\frac{2}{1.667+1} \right) \left(1 + \frac{1.667-1}{2} (0.0912)^2 \right) \right]^{2.667/(2 \times 0.667)} \\ &= 6.20 \end{aligned}$$

Then the ratio of the throat area to the entrance area becomes

$$\frac{A^*}{A_i} = \frac{1}{6.20} = 0.161$$

Discussion The compressible flow functions are essential tools when determining the proper shape of the compressible flow duct.

17-129 Helium gas enters a nozzle with negligible velocity, and is accelerated in a nozzle. The pressure and temperature of helium at the location where $Ma = 1$ and the ratio of the flow area at this location to the inlet flow area are to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The entrance velocity is negligible.

Properties The properties of helium are $R = 2.0769$ kJ/kg·K, $c_p = 5.1926$ kJ/kg·K, and $k = 1.667$ (Table A-2a).

Analysis We treat helium as an ideal gas with $k = 1.667$. The properties of the fluid at the location where $Ma = 1$ are the critical properties, denoted by superscript *.

The stagnation temperature and pressure in this case are identical to the inlet temperature and pressure since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i = 500 \text{ K}$$

$$P_0 = P_i = 0.8 \text{ MPa}$$

The Mach number at the nozzle exit is given to be $Ma = 1$. Therefore, the properties at the nozzle exit are the *critical properties* determined from

$$T^* = T_0 \left(\frac{2}{k+1} \right) = (500 \text{ K}) \left(\frac{2}{1.667+1} \right) = \mathbf{375 \text{ K}}$$

$$P^* = P_0 \left(\frac{2}{k+1} \right)^{k/(k-1)} = (0.8 \text{ MPa}) \left(\frac{2}{1.667+1} \right)^{1.667/(1.667-1)} = \mathbf{0.390 \text{ MPa}}$$

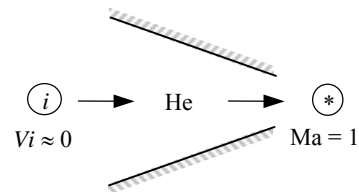
The ratio of the nozzle inlet area to the throat area is determined from

$$\frac{A_i}{A^*} = \frac{1}{Ma_i} \left[\left(\frac{2}{k+1} \right) \left(1 + \frac{k-1}{2} Ma_i^2 \right) \right]^{(k+1)/(2(k-1))}$$

But the Mach number at the nozzle inlet is $Ma = 0$ since $V_i \cong 0$. Thus the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A_i} = \frac{1}{\infty} = \mathbf{0}$$

Discussion The compressible flow functions are essential tools when determining the proper shape of the compressible flow duct.



17-130 EES Air enters a converging nozzle. The mass flow rate, the exit velocity, the exit Mach number, and the exit pressure-stagnation pressure ratio versus the back pressure-stagnation pressure ratio for a specified back pressure range are to be calculated and plotted.

Assumptions 1 Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air at room temperature are $R = 0.287 \text{ kJ/kg}\cdot\text{K}$, $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$, and $k = 1.4$ (Table A-2a).

Analysis The stagnation properties remain constant throughout the nozzle since the flow is isentropic. They are determined from

$$T_0 = T_i + \frac{V_i^2}{2c_p} = 400 \text{ K} + \frac{(180 \text{ m/s})^2}{2 \times 1.005 \text{ kJ/kg}\cdot\text{K}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 416.1 \text{ K}$$

and

$$P_0 = P_i \left(\frac{T_0}{T_i} \right)^{k/(k-1)} = (900 \text{ kPa}) \left(\frac{416.1 \text{ K}}{400 \text{ K}} \right)^{1.4/(1.4-1)} = 1033.3 \text{ kPa}$$

The critical pressure is determined to be

$$P^* = P_0 \left(\frac{2}{k+1} \right)^{k/(k-1)} = (1033.3 \text{ kPa}) \left(\frac{2}{1.4+1} \right)^{1.4/0.4} = 545.9 \text{ kPa}$$

Then the pressure at the exit plane (throat) will be

$$P_e = P_b \quad \text{for } P_b \geq 545.9 \text{ kPa}$$

$$P_e = P^* = 545.9 \text{ kPa} \quad \text{for } P_b < 545.9 \text{ kPa} \text{ (choked flow)}$$

Thus the back pressure will not affect the flow when $100 < P_b < 545.9 \text{ kPa}$. For a specified exit pressure P_e , the temperature, the velocity and the mass flow rate can be determined from

$$\text{Temperature} \quad T_e = T_0 \left(\frac{P_e}{P_0} \right)^{(k-1)/k} = (416.1 \text{ K}) \left(\frac{P_e}{1033.3} \right)^{0.4/1.4}$$

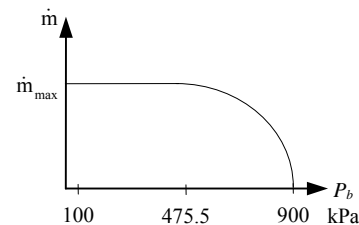
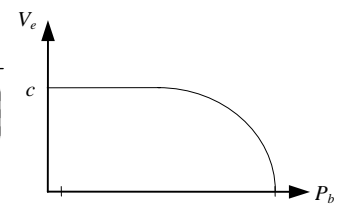
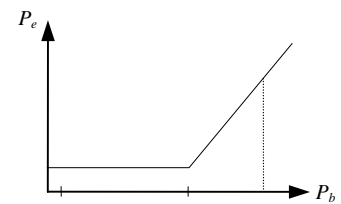
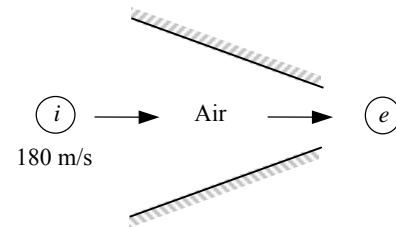
$$\text{Velocity } V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.005 \text{ kJ/kg}\cdot\text{K})(416.1 - T_e) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)}$$

$$\text{Speed of sound} \quad c_e = \sqrt{kRT_e} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)}$$

$$\text{Mach number} \quad \text{Ma}_e = V_e / c_e$$

$$\text{Density} \quad \rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})T_e}$$

$$\text{Mass flow rate} \quad \dot{m} = \rho_e V_e A_e = \rho_e V_e (0.001 \text{ m}^2)$$



The results of the calculations can be tabulated as

$P_b, \text{ kPa}$	P_b, P_0	$P_e, \text{ kPa}$	P_b, P_0	$T_e, \text{ K}$	$V_e, \text{ m/s}$	Ma	$\rho_e, \text{ kg/m}^3$	$\dot{m}, \text{ kg/s}$
900	0.871	900	0.871	400.0	180.0	0.45	7.840	0
800	0.774	800	0.774	386.8	162.9	0.41	7.206	1.174
700	0.677	700	0.677	372.3	236.0	0.61	6.551	1.546
600	0.581	600	0.581	356.2	296.7	0.78	5.869	1.741
545.9	0.528	545.9	0.528	333.3	366.2	1.00	4.971	1.820
500	0.484	545.9	0.528	333.2	366.2	1.00	4.971	1.820
400	0.387	545.9	0.528	333.3	366.2	1.00	4.971	1.820
300	0.290	545.9	0.528	333.3	366.2	1.00	4.971	1.820
200	0.194	545.9	0.528	333.3	366.2	1.00	4.971	1.820
100	0.097	545.9	0.528	333.3	366.2	1.00	4.971	1.820

17-131 EES Steam enters a converging nozzle. The exit pressure, the exit velocity, and the mass flow rate versus the back pressure for a specified back pressure range are to be plotted.

Assumptions **1** Steam is to be treated as an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The ideal gas properties of steam are given to be $R = 0.462 \text{ kJ/kg}\cdot\text{K}$, $c_p = 1.872 \text{ kJ/kg}\cdot\text{K}$, and $k = 1.3$.

Analysis The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Since the flow is isentropic, they remain constant throughout the nozzle,

$$P_0 = P_i = 6 \text{ MPa}$$

$$T_0 = T_i = 700 \text{ K}$$

The critical pressure is determined from to be

$$P^* = P_0 \left(\frac{2}{k+1} \right)^{k/(k-1)} = (6 \text{ MPa}) \left(\frac{2}{1.3+1} \right)^{1.3/0.3} = 3.274 \text{ MPa}$$

Then the pressure at the exit plane (throat) will be

$$P_e = P_b \quad \text{for} \quad P_b \geq 3.274 \text{ MPa}$$

$$P_e = P^* = 3.274 \text{ MPa} \quad \text{for} \quad P_b < 3.274 \text{ MPa} \quad (\text{choked flow})$$

Thus the back pressure will not affect the flow when $3 < P_b < 3.274 \text{ MPa}$. For a specified exit pressure P_e , the temperature, the velocity and the mass flow rate can be determined from

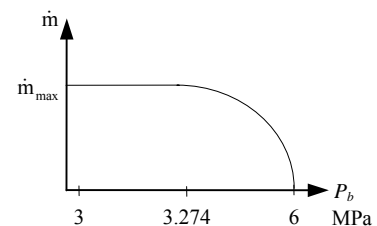
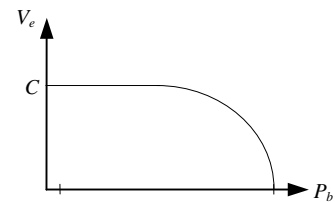
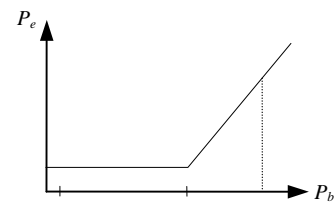
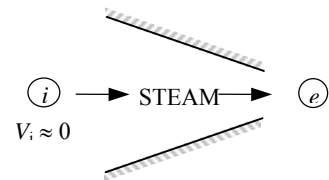
$$\text{Temperature} \quad T_e = T_0 \left(\frac{P_e}{P_0} \right)^{(k-1)/k} = (700 \text{ K}) \left(\frac{P_e}{6} \right)^{0.3/1.3}$$

$$\text{Velocity} \quad V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.872 \text{ kJ/kg}\cdot\text{K})(700 - T_e) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)}$$

$$\text{Density} \quad \rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.462 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})T_e}$$

$$\text{Mass flow rate} \quad \dot{m} = \rho_e V_e A_e = \rho_e V_e (0.0008 \text{ m}^2)$$

The results of the calculations can be tabulated as follows:



$P_b, \text{ MPa}$	$P_e, \text{ MPa}$	$T_e, \text{ K}$	$V_e, \text{ m/s}$	$\rho_e, \text{ kg/m}^3$	$\dot{m}, \text{ kg/s}$
6.0	6.0	700	0	18.55	0
5.5	5.5	686.1	228.1	17.35	3.166
5.0	5.0	671.2	328.4	16.12	4.235
4.5	4.5	655.0	410.5	14.87	4.883
4.0	4.0	637.5	483.7	13.58	5.255
3.5	3.5	618.1	553.7	12.26	5.431
3.274	3.274	608.7	584.7	11.64	5.445
3.0	3.274	608.7	584.7	11.64	5.445

17-132 An expression for the ratio of the stagnation pressure after a shock wave to the static pressure before the shock wave as a function of k and the Mach number upstream of the shock wave is to be found.

Analysis The relation between P_1 and P_2 is

$$\frac{P_2}{P_1} = \frac{1 + k\text{Ma}_2^2}{1 + k\text{Ma}_1^2} \longrightarrow P_2 = P_1 \left(\frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} \right)$$

Substituting this into the isentropic relation

$$\frac{P_{02}}{P_1} = \left(1 + (k-1)\text{Ma}_2^2 / 2 \right)^{k/(k-1)}$$

Then,

$$\frac{P_{02}}{P_1} = \left(\frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} \right) \left(1 + (k-1)\text{Ma}_2^2 / 2 \right)^{k/(k-1)}$$

where

$$\text{Ma}_2^2 = \frac{\text{Ma}_1^2 + 2/(k-1)}{2k\text{Ma}_1^2/(k-1) - 1}$$

Substituting,

$$\frac{P_{02}}{P_1} = \left(\frac{(1 + k\text{Ma}_1^2)(2k\text{Ma}_1^2 - k + 1)}{k\text{Ma}_1^2(k+1) - k + 3} \right) \left(1 + \frac{(k-1)\text{Ma}_1^2 / 2 + 1}{2k\text{Ma}_1^2 / (k-1) - 1} \right)^{k/(k-1)}$$

17-133 Nitrogen entering a converging-diverging nozzle experiences a normal shock. The pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock are to be determined. The results are to be compared to those of air under the same conditions.

Assumptions **1** Nitrogen is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of nitrogen are $R = 0.2968$ kJ/kg·K and $k = 1.4$ (Table A-2a).

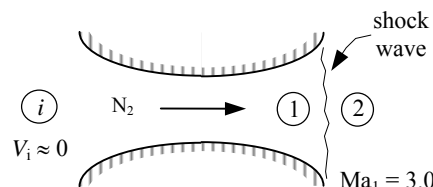
Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Assuming the flow before the shock to be isentropic,

$$P_{01} = P_i = 700 \text{ kPa}$$

$$T_{01} = T_i = 300 \text{ K}$$

Then,

$$T_1 = T_{01} \left(\frac{2}{2 + (k-1)\text{Ma}_1^2} \right) = (300 \text{ K}) \left(\frac{2}{2 + (1.4-1)3^2} \right) = 107.1 \text{ K}$$



and

$$P_1 = P_{01} \left(\frac{T_1}{T_{01}} \right)^{k/(k-1)} = (700 \text{ kPa}) \left(\frac{107.1}{300} \right)^{1.4/0.4} = 19.06 \text{ kPa}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For $\text{Ma}_1 = 3.0$ we read

$$\text{Ma}_2 = \mathbf{0.4752}, \quad \frac{P_{02}}{P_{01}} = 0.32834, \quad \frac{P_2}{P_1} = 10.333, \quad \text{and} \quad \frac{T_2}{T_1} = 2.679$$

Then the stagnation pressure P_{02} , static pressure P_2 , and static temperature T_2 , are determined to be

$$P_{02} = 0.32834 P_{01} = (0.32834)(700 \text{ kPa}) = \mathbf{230 \text{ kPa}}$$

$$P_2 = 10.333 P_1 = (10.333)(19.06 \text{ kPa}) = \mathbf{197 \text{ kPa}}$$

$$T_2 = 2.679 T_1 = (2.679)(107.1 \text{ K}) = \mathbf{287 \text{ K}}$$

The velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2 \sqrt{kRT_2} = (0.4752) \sqrt{(1.4)(0.2968 \text{ kJ/kg} \cdot \text{K})(287 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = \mathbf{164 \text{ m/s}}$$

Discussion For **air** at specified conditions $k = 1.4$ (same as nitrogen) and $R = 0.287$ kJ/kg·K. Thus the only quantity which will be different in the case of air is the velocity after the normal shock, which happens to be 161.3 m/s.

17-134 The diffuser of an aircraft is considered. The static pressure rise across the diffuser and the exit area are to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the diffuser is steady, one-dimensional, and isentropic. **3** The diffuser is adiabatic.

Properties Air properties at room temperature are $R = 0.287 \text{ kJ/kg}\cdot\text{K}$, $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$, and $k = 1.4$ (Table A-2a).

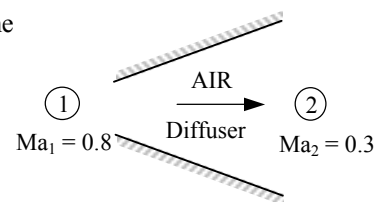
Analysis The inlet velocity is

$$V_1 = \text{Ma}_1 c_1 = M_1 \sqrt{kRT_1} = (0.8) \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(242.7 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 249.8 \text{ m/s}$$

Then the stagnation temperature and pressure at the diffuser inlet become

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 242.7 + \frac{(249.8 \text{ m/s})^2}{2(1.005 \text{ kJ/kg}\cdot\text{K}) \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)} = 273.7 \text{ K}$$

$$P_{01} = P_1 \left(\frac{T_{01}}{T_1} \right)^{k/(k-1)} = (41.1 \text{ kPa}) \left(\frac{273.7 \text{ K}}{242.7 \text{ K}} \right)^{1.4/(1.4-1)} = 62.6 \text{ kPa}$$



For an adiabatic diffuser, the energy equation reduces to $h_{01} = h_{02}$. Noting that $h = c_p T$ and the specific heats are assumed to be constant, we have

$$T_{01} = T_{02} = T_0 = 273.7 \text{ K}$$

The isentropic relation between states 1 and 02 gives

$$P_{02} = P_{01} = P_1 \left(\frac{T_{02}}{T_1} \right)^{k/(k-1)} = (41.1 \text{ kPa}) \left(\frac{273.72 \text{ K}}{242.7 \text{ K}} \right)^{1.4/(1.4-1)} = 62.61 \text{ kPa}$$

The exit velocity can be expressed as

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2 \sqrt{kRT_2} = (0.3) \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K}) T_2 \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 6.01 \sqrt{T_2}$$

$$\text{Thus } T_2 = T_{02} - \frac{V_2^2}{2c_p} = (273.7) - \frac{6.01^2 T_2 \text{ m}^2/\text{s}^2}{2(1.005 \text{ kJ/kg}\cdot\text{K}) \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)} = 268.9 \text{ K}$$

Then the static exit pressure becomes

$$P_2 = P_{02} \left(\frac{T_2}{T_{02}} \right)^{k/(k-1)} = (62.61 \text{ kPa}) \left(\frac{268.9 \text{ K}}{273.7 \text{ K}} \right)^{1.4/(1.4-1)} = 58.85 \text{ kPa}$$

Thus the static pressure rise across the diffuser is

$$\Delta P = P_2 - P_1 = 58.85 - 41.1 = \mathbf{17.8 \text{ kPa}}$$

$$\text{Also, } \rho_2 = \frac{P_2}{RT_2} = \frac{58.85 \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(268.9 \text{ K})} = 0.7626 \text{ kg/m}^3$$

$$V_2 = 6.01 \sqrt{T_2} = 6.01 \sqrt{268.9} = 98.6 \text{ m/s}$$

$$\text{Thus } A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{65 \text{ kg/s}}{(0.7626 \text{ kg/m}^3)(98.6 \text{ m/s})} = \mathbf{0.864 \text{ m}^2}$$

Discussion The pressure rise in actual diffusers will be lower because of the irreversibilities. However, flow through well-designed diffusers is very nearly isentropic.

17-135 Helium gas is accelerated in a nozzle isentropically. For a specified mass flow rate, the throat and exit areas of the nozzle are to be determined.

Assumptions 1 Helium is an ideal gas with constant specific heats. 2 Flow through the nozzle is steady, one-dimensional, and isentropic. 3 The nozzle is adiabatic.

Properties The properties of helium are $R = 2.0769 \text{ kJ/kg}\cdot\text{K}$, $c_p = 5.1926 \text{ kJ/kg}\cdot\text{K}$, $k = 1.667$ (Table A-2a).

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible,

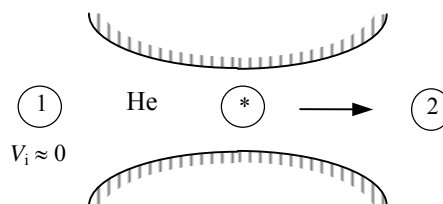
$$T_{01} = T_1 = 500 \text{ K}$$

$$P_{01} = P_1 = 1.0 \text{ MPa}$$

The flow is assumed to be isentropic, thus the stagnation temperature and pressure remain constant throughout the nozzle,

$$T_{02} = T_{01} = 500 \text{ K}$$

$$P_{02} = P_{01} = 1.0 \text{ MPa}$$



The critical pressure and temperature are determined from

$$T^* = T_0 \left(\frac{2}{k+1} \right) = (500 \text{ K}) \left(\frac{2}{1.667+1} \right) = 375.0 \text{ K}$$

$$P^* = P_0 \left(\frac{2}{k+1} \right)^{k/(k-1)} = (1.0 \text{ MPa}) \left(\frac{2}{1.667+1} \right)^{1.667/(1.667-1)} = 0.487 \text{ MPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{487 \text{ kPa}}{(2.0769 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(375 \text{ K})} = 0.625 \text{ kg/m}^3$$

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.667)(2.0769 \text{ kJ/kg}\cdot\text{K})(375 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 1139.4 \text{ m/s}$$

Thus the throat area is

$$A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{0.25 \text{ kg/s}}{(0.625 \text{ kg/m}^3)(1139.4 \text{ m/s})} = 3.51 \times 10^{-4} \text{ m}^2 = \mathbf{3.51 \text{ cm}^2}$$

At the nozzle exit the pressure is $P_2 = 0.1 \text{ MPa}$. Then the other properties at the nozzle exit are determined to be

$$\frac{P_0}{P_2} = \left(1 + \frac{k-1}{2} \text{Ma}_2^2 \right)^{k/(k-1)} \longrightarrow \frac{1.0 \text{ MPa}}{0.1 \text{ MPa}} = \left(1 + \frac{1.667-1}{2} \text{Ma}_2^2 \right)^{1.667/0.667}$$

It yields $\text{Ma}_2 = 2.130$, which is greater than 1. Therefore, the nozzle must be converging-diverging.

$$T_2 = T_0 \left(\frac{2}{2+(k-1)\text{Ma}_2^2} \right) = (500 \text{ K}) \left(\frac{2}{2+(1.667-1)\times 2.13^2} \right) = 199.0 \text{ K}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{100 \text{ kPa}}{(2.0769 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(199 \text{ K})} = 0.242 \text{ kg/m}^3$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2 \sqrt{kRT_2} = (2.13) \sqrt{(1.667)(2.0769 \text{ kJ/kg}\cdot\text{K})(199 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 1768.0 \text{ m/s}$$

Thus the exit area is

$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{0.25 \text{ kg/s}}{(0.242 \text{ kg/m}^3)(1768 \text{ m/s})} = 5.84 \times 10^{-4} \text{ m}^2 = \mathbf{5.84 \text{ cm}^2}$$

Discussion Flow areas in actual nozzles would be somewhat larger to accommodate the irreversibilities.

17-136E Helium gas is accelerated in a nozzle. For a specified mass flow rate, the throat and exit areas of the nozzle are to be determined for the cases of isentropic and 97% efficient nozzles.

Assumptions 1 Helium is an ideal gas with constant specific heats. 2 Flow through the nozzle is steady, one-dimensional, and isentropic. 3 The nozzle is adiabatic.

Properties The properties of helium are $R = 0.4961 \text{ Btu/lbm}\cdot\text{R} = 2.6809 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R}$, $c_p = 1.25 \text{ Btu/lbm}\cdot\text{R}$, and $k = 1.667$ (Table A-2Ea).

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible,

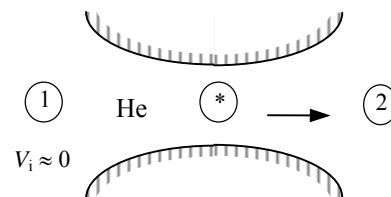
$$T_{01} = T_1 = 900 \text{ R}$$

$$P_{01} = P_1 = 150 \text{ psia}$$

The flow is assumed to be isentropic, thus the stagnation temperature and pressure remain constant throughout the nozzle,

$$T_{02} = T_{01} = 900 \text{ R}$$

$$P_{02} = P_{01} = 150 \text{ psia}$$



The critical pressure and temperature are determined from

$$T^* = T_0 \left(\frac{2}{k+1} \right) = (900 \text{ R}) \left(\frac{2}{1.667+1} \right) = 674.9 \text{ R}$$

$$P^* = P_0 \left(\frac{2}{k+1} \right)^{k/(k-1)} = (150 \text{ psia}) \left(\frac{2}{1.667+1} \right)^{1.667/(1.667-1)} = 73.1 \text{ psia}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{73.1 \text{ psia}}{(2.6809 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(674.9 \text{ R})} = 0.0404 \text{ lbm/ft}^3$$

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.667)(0.4961 \text{ Btu/lbm}\cdot\text{R})(674.9 \text{ R}) \left(\frac{25,037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}} \right)} = 3738 \text{ ft/s}$$

and

$$A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{0.2 \text{ lbm/s}}{(0.0404 \text{ lbm/ft}^3)(3738 \text{ ft/s})} = \mathbf{0.00132 \text{ ft}^2}$$

At the nozzle exit the pressure is $P_2 = 15 \text{ psia}$. Then the other properties at the nozzle exit are determined to be

$$\frac{P_0}{P_2} = \left(1 + \frac{k-1}{2} \text{Ma}_2^2 \right)^{k/(k-1)} \longrightarrow \frac{150 \text{ psia}}{15 \text{ psia}} = \left(1 + \frac{1.667-1}{2} \text{Ma}_2^2 \right)^{1.667/0.667}$$

It yields $\text{Ma}_2 = 2.130$, which is greater than 1. Therefore, the nozzle must be converging-diverging.

$$T_2 = T_0 \left(\frac{2}{2+(k-1)\text{Ma}_2^2} \right) = (900 \text{ R}) \left(\frac{2}{2+(1.667-1)\times 2.13^2} \right) = 358.1 \text{ R}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{15 \text{ psia}}{(2.6809 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(358.1 \text{ R})} = 0.0156 \text{ lbm/ft}^3$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2 \sqrt{kRT_2} = (2.13) \sqrt{(1.667)(0.4961 \text{ Btu/lbm}\cdot\text{R})(358.1 \text{ R}) \left(\frac{25,037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}} \right)} = 5800 \text{ ft/s}$$

Thus the exit area is

$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{0.2 \text{ lbm/s}}{(0.0156 \text{ lbm/ft}^3)(5800 \text{ ft/s})} = \mathbf{0.00221 \text{ ft}^2}$$

Discussion Flow areas in actual nozzles would be somewhat larger to accommodate the irreversibilities.

17-137 [Also solved by EES on enclosed CD] Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-32 for an ideal gas with $k = 1.667$.

Properties The specific heat ratio of the ideal gas is given to be $k = 1.667$.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\text{Ma}^* = \text{Ma} \sqrt{\frac{k+1}{2+(k-1)\text{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}} \left[\left(\frac{2}{k+1} \right) \left(1 + \frac{k-1}{2} \text{Ma}^2 \right) \right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-1}$$

$k=1.667$

$\text{PP0}=(1+(k-1)*\text{M}^2/2)^{-k/(k-1)}$

$\text{TT0}=1/(1+(k-1)*\text{M}^2/2)$

$\text{DD0}=(1+(k-1)*\text{M}^2/2)^{-1/(k-1)}$

$\text{Mcr}=\text{M}*\text{SQRT}((k+1)/(2+(k-1)*\text{M}^2))$

$\text{AAcr}=(2/(k+1))*(1+0.5*(k-1)*\text{M}^2)^{(0.5*(k+1)/(k-1))}/\text{M}$

Ma	Ma*	A/A*	P/P ₀	ρ/ρ ₀	T/T ₀
0.0	0	∞	1.0000	1.0000	1.0000
0.1	0.1153	5.6624	0.9917	0.9950	0.9967
0.2	0.2294	2.8879	0.9674	0.9803	0.9868
0.3	0.3413	1.9891	0.9288	0.9566	0.9709
0.4	0.4501	1.5602	0.8782	0.9250	0.9493
0.5	0.5547	1.3203	0.8186	0.8869	0.9230
0.6	0.6547	1.1760	0.7532	0.8437	0.8928
0.7	0.7494	1.0875	0.6850	0.7970	0.8595
0.8	0.8386	1.0351	0.6166	0.7482	0.8241
0.9	0.9222	1.0081	0.5501	0.6987	0.7873
1.0	1.0000	1.0000	0.4871	0.6495	0.7499
1.2	1.1390	1.0267	0.3752	0.5554	0.6756
1.4	1.2572	1.0983	0.2845	0.4704	0.6047
1.6	1.3570	1.2075	0.2138	0.3964	0.5394
1.8	1.4411	1.3519	0.1603	0.3334	0.4806
2.0	1.5117	1.5311	0.1202	0.2806	0.4284
2.2	1.5713	1.7459	0.0906	0.2368	0.3825
2.4	1.6216	1.9980	0.0686	0.2005	0.3424
2.6	1.6643	2.2893	0.0524	0.1705	0.3073
2.8	1.7007	2.6222	0.0403	0.1457	0.2767
3.0	1.7318	2.9990	0.0313	0.1251	0.2499
5.0	1.8895	9.7920	0.0038	0.0351	0.1071
∞	1.9996	∞	0	0	0

17-138 [Also solved by EES on enclosed CD] Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-33 for an ideal gas with $k = 1.667$.

Properties The specific heat ratio of the ideal gas is given to be $k = 1.667$.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\begin{aligned} \text{Ma}_2 &= \sqrt{\frac{(k-1)\text{Ma}_1^2 + 2}{2k\text{Ma}_1^2 - k + 1}} & \frac{P_2}{P_1} &= \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{2k\text{Ma}_1^2 - k + 1}{k + 1} \\ \frac{T_2}{T_1} &= \frac{2 + \text{Ma}_1^2(k-1)}{2 + \text{Ma}_2^2(k-1)} & \frac{\rho_2}{\rho_1} &= \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\text{Ma}_1^2}{2 + (k-1)\text{Ma}_1^2} = \frac{V_1}{V_2}, \\ \frac{P_{02}}{P_{01}} &= \frac{\text{Ma}_1}{\text{Ma}_2} \left[\frac{1 + \text{Ma}_2^2(k-1)/2}{1 + \text{Ma}_1^2(k-1)/2} \right]^{\frac{k+1}{2(k-1)}} & \frac{P_{02}}{P_1} &= \frac{(1 + k\text{Ma}_1^2)[1 + \text{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\text{Ma}_2^2} \end{aligned}$$

$k=1.667$

$\text{My} = \text{SQRT}((\text{Mx}^2 + 2/(k-1))/(2*\text{Mx}^2*k/(k-1) - 1))$

$\text{PyPx} = (1 + k*\text{Mx}^2)/(1 + k*\text{My}^2)$

$\text{TyTx} = (1 + \text{Mx}^2*(k-1)/2)/(1 + \text{My}^2*(k-1)/2)$

$\text{RyRx} = \text{PyPx}/\text{TyTx}$

$\text{P0yP0x} = (\text{Mx}/\text{My}) * ((1 + \text{My}^2*(k-1)/2)/(1 + \text{Mx}^2*(k-1)/2))^{(0.5*(k+1)/(k-1))}$

$\text{P0yPx} = (1 + k*\text{Mx}^2) * (1 + \text{My}^2*(k-1)/2)^{k/(k-1)} / (1 + k*\text{My}^2)$

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	2.0530
1.1	0.9131	1.2625	1.1496	1.0982	0.999	2.3308
1.2	0.8462	1.5500	1.2972	1.1949	0.9933	2.6473
1.3	0.7934	1.8626	1.4413	1.2923	0.9813	2.9990
1.4	0.7508	2.2001	1.5805	1.3920	0.9626	3.3838
1.5	0.7157	2.5626	1.7141	1.4950	0.938	3.8007
1.6	0.6864	2.9501	1.8415	1.6020	0.9085	4.2488
1.7	0.6618	3.3627	1.9624	1.7135	0.8752	4.7278
1.8	0.6407	3.8002	2.0766	1.8300	0.8392	5.2371
1.9	0.6227	4.2627	2.1842	1.9516	0.8016	5.7767
2.0	0.6070	4.7503	2.2853	2.0786	0.763	6.3462
2.1	0.5933	5.2628	2.3802	2.2111	0.7243	6.9457
2.2	0.5814	5.8004	2.4689	2.3493	0.6861	7.5749
2.3	0.5708	6.3629	2.5520	2.4933	0.6486	8.2339
2.4	0.5614	6.9504	2.6296	2.6432	0.6124	8.9225
2.5	0.5530	7.5630	2.7021	2.7989	0.5775	9.6407
2.6	0.5455	8.2005	2.7699	2.9606	0.5442	10.3885
2.7	0.5388	8.8631	2.8332	3.1283	0.5125	11.1659
2.8	0.5327	9.5506	2.8923	3.3021	0.4824	11.9728
2.9	0.5273	10.2632	2.9476	3.4819	0.4541	12.8091
3.0	0.5223	11.0007	2.9993	3.6678	0.4274	13.6750
4.0	0.4905	19.7514	3.3674	5.8654	0.2374	23.9530
5.0	0.4753	31.0022	3.5703	8.6834	0.1398	37.1723
∞	0.4473	∞	3.9985	∞	0	∞

17-139 The critical temperature, pressure, and density of an equimolar mixture of oxygen and nitrogen for specified stagnation properties are to be determined.

Assumptions Both oxygen and nitrogen are ideal gases with constant specific heats at room temperature.

Properties The specific heat ratio and molar mass are $k = 1.395$ and $M = 32$ kg/kmol for oxygen, and $k = 1.4$ and $M = 28$ kg/kmol for nitrogen (Tables A-1 and A-2).

Analysis The gas constant of the mixture is

$$M_m = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 0.5 \times 32 + 0.5 \times 28 = 30 \text{ kg/kmol}$$

$$R_m = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/kmol} \cdot \text{K}}{30 \text{ kg/kmol}} = 0.2771 \text{ kJ/kg} \cdot \text{K}$$

The specific heat ratio is 1.4 for nitrogen, and nearly 1.4 for oxygen. Therefore, the specific heat ratio of the mixture is also 1.4. Then the critical temperature, pressure, and density of the mixture become

$$T^* = T_0 \left(\frac{2}{k+1} \right) = (800 \text{ K}) \left(\frac{2}{1.4+1} \right) = \mathbf{667 \text{ K}}$$

$$P^* = P_0 \left(\frac{2}{k+1} \right)^{k/(k-1)} = (500 \text{ kPa}) \left(\frac{2}{1.4+1} \right)^{1.4/(1.4-1)} = \mathbf{264 \text{ kPa}}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{264 \text{ kPa}}{(0.2771 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(667 \text{ K})} = \mathbf{1.43 \text{ kg/m}^3}$$

Discussion If the specific heat ratios k of the two gases were different, then we would need to determine the k of the mixture from $k = c_{p,m}/c_{v,m}$ where the specific heats of the mixture are determined from

$$c_{p,m} = \text{mf}_{O_2} c_{p,O_2} + \text{mf}_{N_2} c_{p,N_2} = (y_{O_2} M_{O_2} / M_m) c_{p,O_2} + (y_{N_2} M_{N_2} / M_m) c_{p,N_2}$$

$$c_{v,m} = \text{mf}_{O_2} c_{v,O_2} + \text{mf}_{N_2} c_{v,N_2} = (y_{O_2} M_{O_2} / M_m) c_{v,O_2} + (y_{N_2} M_{N_2} / M_m) c_{v,N_2}$$

where mf is the mass fraction and y is the mole fraction. In this case it would give

$$c_{p,m} = (0.5 \times 32 / 30) \times 0.918 + (0.5 \times 28 / 30) \times 1.039 = 0.974 \text{ kJ/kg} \cdot \text{K}$$

$$c_{v,m} = (0.5 \times 32 / 30) \times 0.658 + (0.5 \times 28 / 30) \times 0.743 = 0.698 \text{ kJ/kg} \cdot \text{K}$$

and $k = 0.974/0.698 = 1.40$

17-140 EES Using EES (or other) software, the shape of a converging-diverging nozzle is to be determined for specified flow rate and stagnation conditions. The nozzle and the Mach number are to be plotted.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The specific heat ratio of air at room temperature is 1.4 (Table A-2a).

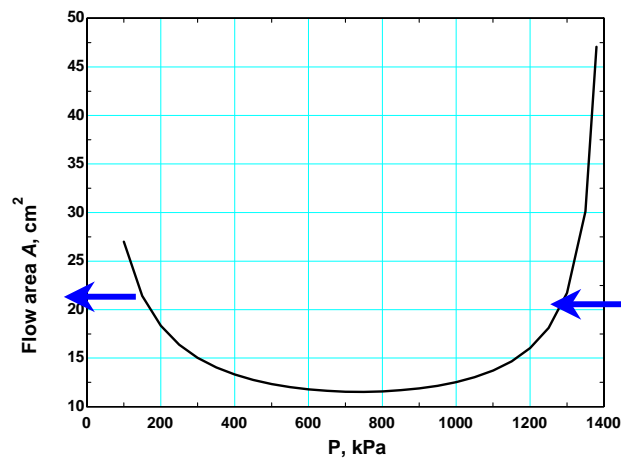
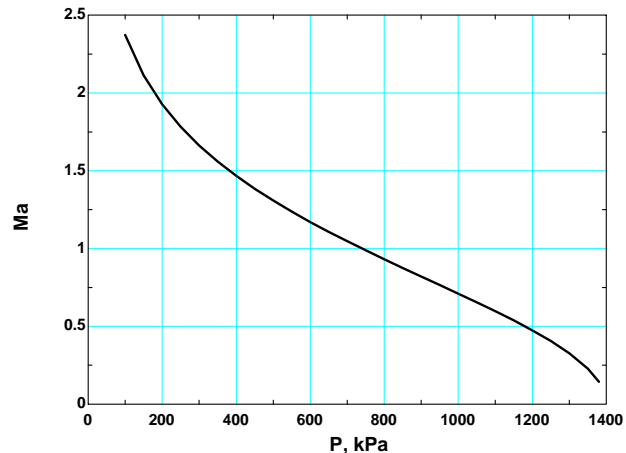
Analysis The problems is solved using EES, and the results are tabulated and plotted below.

```

k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
P0=1400 "kPa"
T0=200+273 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
T=T0*(P/P0)^((k-1)/k)
V=SQRT(2*Cp*(T0-T)*1000)
A=m/(rho*V)*10000 "cm2"
C=SQRT(k*R*T*1000)
Ma=V/C

```

Pressure P_2 , kPa	Flow area A , cm^2	Mach number Ma
1400	∞	0
1350	30.1	0.229
1300	21.7	0.327
1250	18.1	0.406
1200	16.0	0.475
1150	14.7	0.538
1100	13.7	0.597
1050	13.0	0.655
1000	12.5	0.710
950	12.2	0.766
900	11.9	0.820
850	11.7	0.876
800	11.6	0.931
750	11.5	0.988
700	11.5	1.047
650	11.6	1.107
600	11.8	1.171
550	12.0	1.237
500	12.3	1.308
450	12.8	1.384
400	13.3	1.467
350	14.0	1.559
300	15.0	1.663
250	16.4	1.784
200	18.3	1.929
150	21.4	2.114
100	27.0	2.373



17-141 EES Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-32 for air.

Properties The specific heat ratio is given to be $k = 1.4$ for air

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\text{Ma}^* = \text{Ma} \sqrt{\frac{k+1}{2+(k-1)\text{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}} \left[\left(\frac{2}{k+1} \right) \left(1 + \frac{k-1}{2} \text{Ma}^2 \right) \right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-1}$$

Air:

$k=1.4$

$\text{PP0}=(1+(k-1)*\text{M}^2/2)^{-k/(k-1)}$

$\text{TT0}=1/(1+(k-1)*\text{M}^2/2)$

$\text{DD0}=(1+(k-1)*\text{M}^2/2)^{-1/(k-1)}$

$\text{Mcr}=\text{M}*\text{SQRT}((k+1)/(2+(k-1)*\text{M}^2))$

$\text{AAcr}=(2/(k+1))*(1+0.5*(k-1)*\text{M}^2)^{(0.5*(k+1)/(k-1))}/\text{M}$

Ma	Ma*	A/A*	P/P ₀	ρ/ρ ₀	T/T ₀
1.0	1.0000	1.0000	0.5283	0.6339	0.8333
1.5	1.3646	1.1762	0.2724	0.3950	0.6897
2.0	1.6330	1.6875	0.1278	0.2300	0.5556
2.5	1.8257	2.6367	0.0585	0.1317	0.4444
3.0	1.9640	4.2346	0.0272	0.0762	0.3571
3.5	2.0642	6.7896	0.0131	0.0452	0.2899
4.0	2.1381	10.7188	0.0066	0.0277	0.2381
4.5	2.1936	16.5622	0.0035	0.0174	0.1980
5.0	2.2361	25.0000	0.0019	0.0113	0.1667
5.5	2.2691	36.8690	0.0011	0.0076	0.1418
6.0	2.2953	53.1798	0.0006	0.0052	0.1220
6.5	2.3163	75.1343	0.0004	0.0036	0.1058
7.0	2.3333	104.1429	0.0002	0.0026	0.0926
7.5	2.3474	141.8415	0.0002	0.0019	0.0816
8.0	2.3591	190.1094	0.0001	0.0014	0.0725
8.5	2.3689	251.0862	0.0001	0.0011	0.0647
9.0	2.3772	327.1893	0.0000	0.0008	0.0581
9.5	2.3843	421.1314	0.0000	0.0006	0.0525
10.0	2.3905	535.9375	0.0000	0.0005	0.0476

17-142 EES Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-32 for methane.

Properties The specific heat ratio is given to be $k = 1.3$ for methane.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\text{Ma}^* = \text{Ma} \sqrt{\frac{k+1}{2+(k-1)\text{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}} \left[\left(\frac{2}{k+1} \right) \left(1 + \frac{k-1}{2} \text{Ma}^2 \right) \right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-1}$$

Methane:

$k=1.3$

$\text{PP0}=(1+(k-1)*\text{M}^2/2)^{-k/(k-1)}$

$\text{TT0}=1/(1+(k-1)*\text{M}^2/2)$

$\text{DD0}=(1+(k-1)*\text{M}^2/2)^{-1/(k-1)}$

$\text{Mcr}=\text{M}*\text{SQRT}((k+1)/(2+(k-1)*\text{M}^2))$

$\text{AAcr}=(2/(k+1))*(1+0.5*(k-1)*\text{M}^2)^{(0.5*(k+1)/(k-1))}/\text{M}$

Ma	Ma*	A/A*	P/P ₀	ρ/ρ ₀	T/T ₀
1.0	1.0000	1.0000	0.5457	0.6276	0.8696
1.5	1.3909	1.1895	0.2836	0.3793	0.7477
2.0	1.6956	1.7732	0.1305	0.2087	0.6250
2.5	1.9261	2.9545	0.0569	0.1103	0.5161
3.0	2.0986	5.1598	0.0247	0.0580	0.4255
3.5	2.2282	9.1098	0.0109	0.0309	0.3524
4.0	2.3263	15.9441	0.0050	0.0169	0.2941
4.5	2.4016	27.3870	0.0024	0.0095	0.2477
5.0	2.4602	45.9565	0.0012	0.0056	0.2105
5.5	2.5064	75.2197	0.0006	0.0033	0.1806
6.0	2.5434	120.0965	0.0003	0.0021	0.1563
6.5	2.5733	187.2173	0.0002	0.0013	0.1363
7.0	2.5978	285.3372	0.0001	0.0008	0.1198
7.5	2.6181	425.8095	0.0001	0.0006	0.1060
8.0	2.6350	623.1235	0.0000	0.0004	0.0943
8.5	2.6493	895.5077	0.0000	0.0003	0.0845
9.0	2.6615	1265.6040	0.0000	0.0002	0.0760
9.5	2.6719	1761.2133	0.0000	0.0001	0.0688
10.0	2.6810	2416.1184	0.0000	0.0001	0.0625

17-143 EES Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-33 for air.

Properties The specific heat ratio is given to be $k = 1.4$ for air.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\begin{aligned} \text{Ma}_2 &= \sqrt{\frac{(k-1)\text{Ma}_1^2 + 2}{2k\text{Ma}_1^2 - k + 1}} & \frac{P_2}{P_1} &= \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{2k\text{Ma}_1^2 - k + 1}{k + 1} \\ \frac{T_2}{T_1} &= \frac{2 + \text{Ma}_1^2(k-1)}{2 + \text{Ma}_2^2(k-1)} & \frac{\rho_2}{\rho_1} &= \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\text{Ma}_1^2}{2 + (k-1)\text{Ma}_1^2} = \frac{V_1}{V_2}, \\ \frac{P_{02}}{P_{01}} &= \frac{\text{Ma}_1}{\text{Ma}_2} \left[\frac{1 + \text{Ma}_2^2(k-1)/2}{1 + \text{Ma}_1^2(k-1)/2} \right]^{k/(k-1)} & \frac{P_{02}}{P_1} &= \frac{(1 + k\text{Ma}_1^2)[1 + \text{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\text{Ma}_2^2} \end{aligned}$$

Air:

$k=1.4$

$\text{My} = \text{SQRT}((\text{Mx}^2 + 2/(k-1))/(2*\text{Mx}^2*k/(k-1) - 1))$

$\text{PyPx} = (1 + k*\text{Mx}^2)/(1 + k*\text{My}^2)$

$\text{TyTx} = (1 + \text{Mx}^2*(k-1)/2)/(1 + \text{My}^2*(k-1)/2)$

$\text{RyRx} = \text{PyPx}/\text{TyTx}$

$\text{P0yP0x} = (\text{Mx}/\text{My}) * ((1 + \text{My}^2*(k-1)/2)/(1 + \text{Mx}^2*(k-1)/2))^{0.5*(k+1)/(k-1)}$

$\text{P0yPx} = (1 + k*\text{Mx}^2) * (1 + \text{My}^2*(k-1)/2)^{k/(k-1)} / (1 + k*\text{My}^2)$

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	1.8929
1.5	0.7011	2.4583	1.8621	1.3202	0.9298	3.4133
2.0	0.5774	4.5000	2.6667	1.6875	0.7209	5.6404
2.5	0.5130	7.1250	3.3333	2.1375	0.499	8.5261
3.0	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610
3.5	0.4512	14.1250	4.2609	3.3151	0.2129	16.2420
4.0	0.4350	18.5000	4.5714	4.0469	0.1388	21.0681
4.5	0.4236	23.4583	4.8119	4.8751	0.0917	26.5387
5.0	0.4152	29.0000	5.0000	5.8000	0.06172	32.6535
5.5	0.4090	35.1250	5.1489	6.8218	0.04236	39.4124
6.0	0.4042	41.8333	5.2683	7.9406	0.02965	46.8152
6.5	0.4004	49.1250	5.3651	9.1564	0.02115	54.8620
7.0	0.3974	57.0000	5.4444	10.4694	0.01535	63.5526
7.5	0.3949	65.4583	5.5102	11.8795	0.01133	72.8871
8.0	0.3929	74.5000	5.5652	13.3867	0.008488	82.8655
8.5	0.3912	84.1250	5.6117	14.9911	0.006449	93.4876
9.0	0.3898	94.3333	5.6512	16.6927	0.004964	104.7536
9.5	0.3886	105.1250	5.6850	18.4915	0.003866	116.6634
10.0	0.3876	116.5000	5.7143	20.3875	0.003045	129.2170

17-144 EES Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-14 for methane.

Properties The specific heat ratio is given to be $k = 1.3$ for methane.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\begin{aligned} \text{Ma}_2 &= \sqrt{\frac{(k-1)\text{Ma}_1^2 + 2}{2k\text{Ma}_1^2 - k + 1}} & \frac{P_2}{P_1} &= \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{2k\text{Ma}_1^2 - k + 1}{k + 1} \\ \frac{T_2}{T_1} &= \frac{2 + \text{Ma}_1^2(k-1)}{2 + \text{Ma}_2^2(k-1)} & \frac{\rho_2}{\rho_1} &= \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\text{Ma}_1^2}{2 + (k-1)\text{Ma}_1^2} = \frac{V_1}{V_2}, \\ \frac{P_{02}}{P_{01}} &= \frac{\text{Ma}_1}{\text{Ma}_2} \left[\frac{1 + \text{Ma}_2^2(k-1)/2}{1 + \text{Ma}_1^2(k-1)/2} \right]^{k/(k-1)} & \frac{P_{02}}{P_1} &= \frac{(1 + k\text{Ma}_1^2)[1 + \text{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\text{Ma}_2^2} \end{aligned}$$

Methane:

$k=1.3$

$\text{My} = \text{SQRT}((\text{Mx}^2 + 2/(k-1))/(2*\text{Mx}^2*k/(k-1) - 1))$

$\text{PyPx} = (1 + k*\text{Mx}^2)/(1 + k*\text{My}^2)$

$\text{TyTx} = (1 + \text{Mx}^2*(k-1)/2)/(1 + \text{My}^2*(k-1)/2)$

$\text{RyRx} = \text{PyPx}/\text{TyTx}$

$\text{P0yP0x} = (\text{Mx}/\text{My}) * ((1 + \text{My}^2*(k-1)/2)/(1 + \text{Mx}^2*(k-1)/2))^{0.5*(k+1)/(k-1)}$

$\text{P0yPx} = (1 + k*\text{Mx}^2) * (1 + \text{My}^2*(k-1)/2)^{k/(k-1)} / (1 + k*\text{My}^2)$

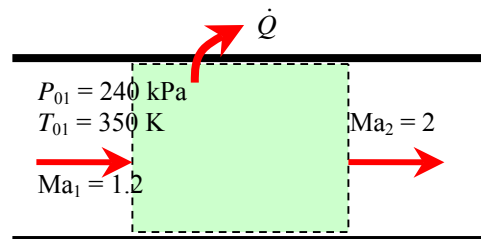
Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	1.8324
1.5	0.6942	2.4130	1.9346	1.2473	0.9261	3.2654
2.0	0.5629	4.3913	2.8750	1.5274	0.7006	5.3700
2.5	0.4929	6.9348	3.7097	1.8694	0.461	8.0983
3.0	0.4511	10.0435	4.4043	2.2804	0.2822	11.4409
3.5	0.4241	13.7174	4.9648	2.7630	0.1677	15.3948
4.0	0.4058	17.9565	5.4118	3.3181	0.09933	19.9589
4.5	0.3927	22.7609	5.7678	3.9462	0.05939	25.1325
5.0	0.3832	28.1304	6.0526	4.6476	0.03613	30.9155
5.5	0.3760	34.0652	6.2822	5.4225	0.02243	37.3076
6.0	0.3704	40.5652	6.4688	6.2710	0.01422	44.3087
6.5	0.3660	47.6304	6.6218	7.1930	0.009218	51.9188
7.0	0.3625	55.2609	6.7485	8.1886	0.006098	60.1379
7.5	0.3596	63.4565	6.8543	9.2579	0.004114	68.9658
8.0	0.3573	72.2174	6.9434	10.4009	0.002827	78.4027
8.5	0.3553	81.5435	7.0190	11.6175	0.001977	88.4485
9.0	0.3536	91.4348	7.0837	12.9079	0.001404	99.1032
9.5	0.3522	101.8913	7.1393	14.2719	0.001012	110.367
10.0	0.3510	112.9130	7.1875	15.7096	0.000740	122.239

17-145 Air flowing at a supersonic velocity in a duct is accelerated by cooling. For a specified exit Mach number, the rate of heat transfer is to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$, and $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ (Table A-2a).

Analysis Knowing stagnation properties, the static properties are determined to be



$$T_1 = T_{01} \left(1 + \frac{k-1}{2} \text{Ma}_1^2 \right)^{-1} = (350 \text{ K}) \left(1 + \frac{1.4-1}{2} 1.2^2 \right)^{-1} = 271.7 \text{ K}$$

$$P_1 = P_{01} \left(1 + \frac{k-1}{2} \text{Ma}_1^2 \right)^{-k/(k-1)} = (240 \text{ kPa}) \left(1 + \frac{1.4-1}{2} 1.2^2 \right)^{-1.4/0.4} = 98.97 \text{ kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{98.97 \text{ kPa}}{(0.287 \text{ kJ/kg}\cdot\text{K})(271.7 \text{ K})} = 1.269 \text{ kg/m}^3$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(271.7 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 330.4 \text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 1.2(330.4 \text{ m/s}) = 396.5 \text{ m/s}$$

$$\dot{m}_{\text{air}} = \rho_1 A c_1 V_1 = (1.269 \text{ kg/m}^3) [\pi(0.20 \text{ m})^2 / 4] (330.4 \text{ m/s}) = 15.81 \text{ kg/s}$$

The Rayleigh flow functions T_0/T_0^* corresponding to the inlet and exit Mach numbers are (Table A-34):

$$\text{Ma}_1 = 1.2: \quad T_{01}/T_0^* = 0.9787$$

$$\text{Ma}_2 = 2: \quad T_{02}/T_0^* = 0.7934$$

Then the exit stagnation temperature is determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{0.7934}{0.9787} = 0.8107 \quad \rightarrow \quad T_{02} = 0.8107 T_{01} = 0.8107(350 \text{ K}) = 283.7 \text{ K}$$

Finally, the rate of heat transfer is

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (15.81 \text{ kg/s})(1.005 \text{ kJ/kg}\cdot\text{K})(283.7 - 350) \text{ K} = \mathbf{-1053 \text{ kW}}$$

Discussion The negative sign confirms that the gas needs to be cooled in order to be accelerated. Also, it can be shown that the thermodynamic temperature drops to 158 K at the exit, which is extremely low. Therefore, the duct may need to be heavily insulated to maintain indicated flow conditions.

17-146 Air flowing at a subsonic velocity in a duct is accelerated by heating. The highest rate of heat transfer without affecting the inlet conditions is to be determined.

Assumptions 1 The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K (Table A-2a).

Analysis Heat transfer will stop when the flow is choked, and thus $Ma_2 = V_2/c_2 = 1$. The inlet density and stagnation temperature are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{400 \text{ kPa}}{(0.287 \text{ kJ/kg}\cdot\text{K})(360 \text{ K})} = 3.872 \text{ kg/m}^3$$

$$T_{01} = T_1 \left(1 + \frac{k-1}{2} Ma_1^2 \right) = (360 \text{ K}) \left(1 + \frac{1.4-1}{2} 0.4^2 \right) = 371.5 \text{ K}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(360 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 380.3 \text{ m/s}$$

$$V_1 = Ma_1 c_1 = 0.4(380.3 \text{ m/s}) = 152.1 \text{ m/s}$$

$$\dot{m}_{\text{air}} = \rho_1 A_{c1} V_1 = (3.872 \text{ kg/m}^3)(0.1 \times 0.1 \text{ m}^2)(152.1 \text{ m/s}) = 5.890 \text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$$T_{02}/T_0^* = 1 \quad (\text{since } Ma_2 = 1)$$

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)Ma_1^2 [2 + (k-1)Ma_1^2]}{(1+kMa_1^2)^2} = \frac{(1.4+1)0.4^2 [2 + (1.4-1)0.4^2]}{(1+1.4 \times 0.4^2)^2} = 0.5290$$

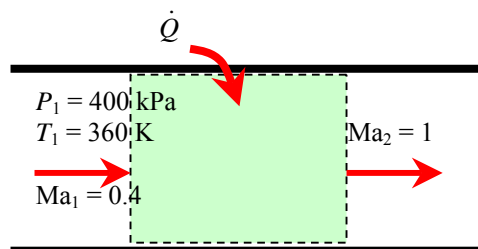
Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.5290} \quad \rightarrow \quad T_{02} = T_{01} / 0.5290 = (371.5 \text{ K}) / 0.5290 = 702.3 \text{ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (5.890 \text{ kg/s})(1.005 \text{ kJ/kg}\cdot\text{K})(702.3 - 371.5) \text{ K} = \mathbf{1958 \text{ kW}}$$

Discussion It can also be shown that $T_2 = 585$ K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. We can also solve this problem using the Rayleigh function values listed in Table A-34.



17-147 Helium flowing at a subsonic velocity in a duct is accelerated by heating. The highest rate of heat transfer without affecting the inlet conditions is to be determined.

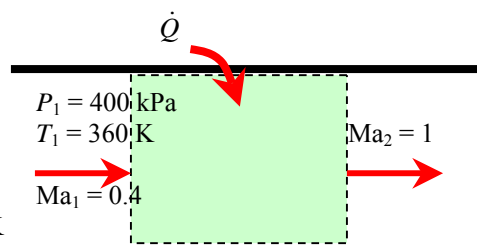
Assumptions 1 The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of helium to be $k = 1.667$, $c_p = 5.193$ kJ/kg·K, and $R = 2.077$ kJ/kg·K (Table A-2a).

Analysis Heat transfer will stop when the flow is choked, and thus $Ma_2 = V_2/c_2 = 1$. The inlet density and stagnation temperature are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{400 \text{ kPa}}{(2.077 \text{ kJ/kg}\cdot\text{K})(360 \text{ K})} = 0.5350 \text{ kg/m}^3$$

$$T_{01} = T_1 \left(1 + \frac{k-1}{2} Ma_1^2 \right) = (360 \text{ K}) \left(1 + \frac{1.667-1}{2} 0.4^2 \right) = 379.2 \text{ K}$$



Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.667)(2.077 \text{ kJ/kg}\cdot\text{K})(360 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 1116 \text{ m/s}$$

$$V_1 = Ma_1 c_1 = 0.4(1116 \text{ m/s}) = 446.6 \text{ m/s}$$

$$\dot{m}_{\text{air}} = \rho_1 A_{c1} V_1 = (0.5350 \text{ kg/m}^3)(0.1 \times 0.1 \text{ m}^2)(446.6 \text{ m/s}) = 2.389 \text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$$T_{02}/T_0^* = 1 \quad (\text{since } Ma_2 = 1)$$

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)Ma_1^2 [2 + (k-1)Ma_1^2]}{(1+kMa_1^2)^2} = \frac{(1.667+1)0.4^2 [2 + (1.667-1)0.4^2]}{(1+1.667 \times 0.4^2)^2} = 0.5603$$

Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.5603} \quad \rightarrow \quad T_{02} = T_{01} / 0.5603 = (379.2 \text{ K}) / 0.5603 = 676.8 \text{ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (2.389 \text{ kg/s})(5.193 \text{ kJ/kg}\cdot\text{K})(676.8 - 379.2) \text{ K} = \mathbf{3693 \text{ kW}}$$

Discussion It can also be shown that $T_2 = 508$ K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. Also, in the solution of this problem, we cannot use the values of Table A-34 since they are based on $k = 1.4$.

17-148 Air flowing at a subsonic velocity in a duct is accelerated by heating. For a specified exit Mach number, the heat transfer for a specified exit Mach number as well as the maximum heat transfer are to be determined.

Assumptions 1 The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

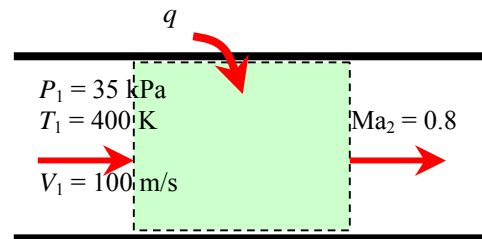
Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$, and $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ (Table A-2a).

Analysis The inlet Mach number and stagnation temperature are

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(400 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 400.9 \text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{100 \text{ m/s}}{400.9 \text{ m/s}} = 0.2494$$

$$\begin{aligned} T_{01} &= T_1 \left(1 + \frac{k-1}{2} \text{Ma}_1^2\right) \\ &= (400 \text{ K}) \left(1 + \frac{1.4-1}{2} (0.2494)^2\right) \\ &= 405.0 \text{ K} \end{aligned}$$



The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-34):

$$\text{Ma}_1 = 0.2494: \quad T_{01}/T^* = 0.2559$$

$$\text{Ma}_2 = 0.8: \quad T_{02}/T^* = 0.9639$$

Then the exit stagnation temperature and the heat transfer are determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{0.9639}{0.2559} = 3.7667 \rightarrow T_{02} = 3.7667 T_{01} = 3.7667(405.0 \text{ K}) = 1526 \text{ K}$$

$$q = c_p(T_{02} - T_{01}) = (1.005 \text{ kJ/kg}\cdot\text{K})(1526 - 405) \text{ K} = \mathbf{1126 \text{ kJ/kg}}$$

Maximum heat transfer will occur when the flow is choked, and thus $\text{Ma}_2 = 1$ and thus $T_{02}/T^* = 1$. Then,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.2559} \rightarrow T_{02} = T_{01} / 0.2559 = (405.0 \text{ K}) / 0.2559 = 1583 \text{ K}$$

$$q_{\max} = c_p(T_{02} - T_{01}) = (1.005 \text{ kJ/kg}\cdot\text{K})(1583 - 405) \text{ K} = \mathbf{1184 \text{ kJ/kg}}$$

Discussion This is the maximum heat that can be transferred to the gas without affecting the mass flow rate. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease.

17-149 Air flowing at sonic conditions in a duct is accelerated by cooling. For a specified exit Mach number, the amount of heat transfer per unit mass is to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

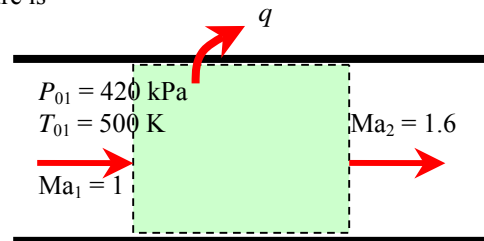
Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$, and $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ (Table A-2a).

Analysis Noting that $\text{Ma}_1 = 1$, the inlet stagnation temperature is

$$T_{01} = T_1 \left(1 + \frac{k-1}{2} \text{Ma}_1^2 \right) = (500 \text{ K}) \left(1 + \frac{1.4-1}{2} 1^2 \right) = 600 \text{ K}$$

The Rayleigh flow functions T_0/T_0^* corresponding to the inlet and exit Mach numbers are (Table A-34):

$$\begin{aligned} \text{Ma}_1 = 1: & \quad T_{01}/T_0^* = 1 \\ \text{Ma}_2 = 1.6: & \quad T_{02}/T_0^* = 0.8842 \end{aligned}$$



Then the exit stagnation temperature and heat transfer are determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{0.8842}{1} = 0.8842 \quad \rightarrow \quad T_{02} = 0.8842 T_{01} = 0.8842(600 \text{ K}) = 530.5 \text{ K}$$

$$q = c_p (T_{02} - T_{01}) = (1.005 \text{ kJ/kg}\cdot\text{K})(530.5 - 600) \text{ K} = -69.8 \text{ kJ/kg}$$

Discussion The negative sign confirms that the gas needs to be cooled in order to be accelerated. Also, it can be shown that the thermodynamic temperature drops to 351 K at the exit

17-150 Saturated steam enters a converging-diverging nozzle with a low velocity. The throat area, exit velocity, mass flow rate, and exit Mach number are to be determined for isentropic and 90 percent efficient nozzle cases.

Assumptions **1** Flow through the nozzle is steady and one-dimensional. **2** The nozzle is adiabatic.

Analysis (a) The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{10} = h_1$. At the inlet,

$$h_1 = (h_f + x_1 h_{fg})_{@3 \text{ MPa}} = 1008.3 + 0.95 \times 1794.9 = 2713.4 \text{ kJ/kg}$$

$$s_1 = (s_f + x_1 s_{fg})_{@3 \text{ MPa}} = 2.6454 + 0.95 \times 3.5402 = 6.0086 \text{ kJ/kg} \cdot \text{K}$$

At the exit, $P_2 = 1.2 \text{ MPa}$ and $s_2 = s_{2s} = s_1 = 6.0086 \text{ kJ/kg} \cdot \text{K}$. Thus,

$$s_2 = s_f + x_2 s_{fg} \rightarrow 6.0086 = 2.2159 + x_2 (4.3058) \rightarrow x_2 = 0.8808$$

$$h_2 = h_f + x_2 h_{fg} = 798.33 + 0.8808 \times 1985.4 = 2547.2 \text{ kJ/kg}$$

$$v_2 = v_f + x_2 v_{fg} = 0.001138 + 0.8808 \times (0.16326 - 0.001138) = 0.1439 \text{ m}^3/\text{kg}$$

Then the exit velocity is determined from the steady-flow energy balance to be

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \rightarrow 0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

Solving for V_2 ,

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(2713.4 - 2547.2) \text{ kJ/kg} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = \mathbf{576.7 \text{ m/s}}$$

The mass flow rate is determined from

$$\dot{m} = \frac{1}{v_2} A_2 V_2 = \frac{1}{0.1439 \text{ m}^3/\text{kg}} (16 \times 10^{-4} \text{ m}^2) (576.7 \text{ m/s}) = \mathbf{6.41 \text{ kg/s}}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial \mathcal{P}}{\partial v} \right)_s \cong \left(\frac{\Delta \mathcal{P}}{\Delta(1/v)} \right)_s^{1/2}$$

The specific volume of steam at $s_2 = 6.0086 \text{ kJ/kg} \cdot \text{K}$ and at pressures just below and just above the specified pressure (1.1 and 1.3 MPa) are determined to be 0.1555 and 0.1340 m^3/kg . Substituting,

$$c_2 = \sqrt{\frac{(1300 - 1100) \text{ kPa}}{\left(\frac{1}{0.1340} - \frac{1}{0.1555} \right) \text{ kg/m}^3} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kPa} \cdot \text{m}^3} \right)} = 440.3 \text{ m/s}$$

Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{576.7 \text{ m/s}}{440.3 \text{ m/s}} = \mathbf{1.310}$$

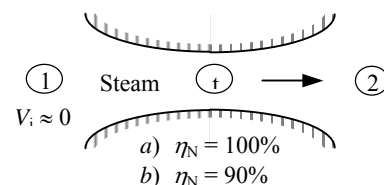
The steam is saturated, and thus the critical pressure which occurs at the throat is taken to be

$$P_t = P^* = 0.576 \times P_{01} = 0.576 \times 3 = 1.728 \text{ MPa}$$

Then at the throat,

$$P_t = 1.728 \text{ MPa} \text{ and } s_t = s_1 = 6.0086 \text{ kJ/kg} \cdot \text{K}$$

Thus,



$$h_t = 2611.4 \text{ kJ/kg}$$

$$\nu_t = 0.1040 \text{ m}^3/\text{kg}$$

Then the throat velocity is determined from the steady-flow energy balance,

$$h_1 + \frac{V_1^2}{2} = h_t + \frac{V_t^2}{2} \rightarrow 0 = h_t - h_1 + \frac{V_t^2}{2}$$

Solving for V_t ,

$$V_t = \sqrt{2(h_1 - h_t)} = \sqrt{2(2713.4 - 2611.4) \text{ kJ/kg} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 451.7 \text{ m/s}$$

Thus the throat area is

$$A_t = \frac{\dot{m} \nu_t}{V_t} = \frac{(6.41 \text{ kg/s})(0.1040 \text{ m}^3/\text{kg})}{(451.7 \text{ m/s})} = 14.75 \times 10^{-4} \text{ m}^2 = \mathbf{14.75 \text{ cm}^2}$$

(b) The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{10} = h_1$. At the inlet,

$$h_1 = (h_f + x_1 h_{fg})_{@3 \text{ MPa}} = 1008.3 + 0.95 \times 1794.9 = 2713.4 \text{ kJ/kg}$$

$$s_1 = (s_f + x_1 s_{fg})_{@3 \text{ MPa}} = 2.6454 + 0.95 \times 3.5402 = 6.0086 \text{ kJ/kg} \cdot \text{K}$$

At state 2s, $P_2 = 1.2 \text{ MPa}$ and $s_2 = s_{2s} = s_1 = 6.0086 \text{ kJ/kg} \cdot \text{K}$. Thus,

$$s_{2s} = s_f + x_{2s} s_{fg} \rightarrow 6.0086 = 2.2159 + x_{2s} (4.3058) \rightarrow x_{2s} = 0.8808$$

$$h_{2s} = h_f + x_{2s} h_{fg} = 798.33 + 0.8808 \times 1985.4 = 2547.2 \text{ kJ/kg}$$

The enthalpy of steam at the actual exit state is determined from

$$\eta_N = \frac{h_{01} - h_2}{h_{01} - h_{2s}} \rightarrow 0.90 = \frac{2713.4 - h_2}{2713.4 - 2547.2} \rightarrow h_2 = 2563.8 \text{ kJ/kg}$$

Therefore at the exit, $P_2 = 1.2 \text{ MPa}$ and $h_2 = 2563.8 \text{ kJ/kg} \cdot \text{K}$. Thus,

$$h_2 = h_f + x_2 h_{fg} \rightarrow 2563.8 = 798.33 + x_2 (1985.4) \rightarrow x_2 = 0.8892$$

$$s_2 = s_f + x_2 s_{fg} = 2.2159 + 0.8892 \times 4.3058 = 6.0447$$

$$\nu_2 = \nu_f + x_2 \nu_{fg} = 0.001138 + 0.8892 \times (0.16326 - 0.001138) = 0.1453 \text{ kJ/kg}$$

Then the exit velocity is determined from the steady-flow energy balance to be

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \rightarrow 0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

Solving for V_2 ,

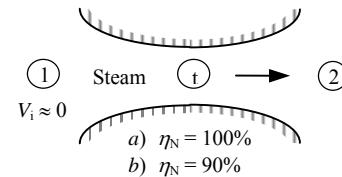
$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(2713.4 - 2563.8) \text{ kJ/kg} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = \mathbf{547.1 \text{ m/s}}$$

The mass flow rate is determined from

$$\dot{m} = \frac{1}{\nu_2} A_2 V_2 = \frac{1}{0.1453 \text{ m}^3/\text{kg}} (16 \times 10^{-4} \text{ m}^2) (547.1 \text{ m/s}) = \mathbf{6.02 \text{ kg/s}}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial P}{\partial \rho} \right)_s^{1/2} \cong \left(\frac{\Delta P}{\Delta(1/\nu)} \right)_s^{1/2}$$



The specific volume of steam at $s_2 = 6.0447$ kJ/kg·K and at pressures just below and just above the specified pressure (1.1 and 1.3 MPa) are determined to be 0.1570 and 0.1353 m³/kg. Substituting,

$$c_2 = \sqrt{\frac{(1300 - 1100) \text{ kPa}}{\left(\frac{1}{0.1353} - \frac{1}{0.1570}\right) \text{ kg/m}^3} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kPa} \cdot \text{m}^3}\right)} = 442.6 \text{ m/s}$$

Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{547.1 \text{ m/s}}{442.6 \text{ m/s}} = \mathbf{1.236}$$

The steam is saturated, and thus the critical pressure which occurs at the throat is taken to be

$$P_t = P^* = 0.576 \times P_{01} = 0.576 \times 3 = 1.728 \text{ MPa}$$

At state 2ts, $P_{ts} = 1.728$ MPa and $s_{ts} = s_1 = 6.0086$ kJ/kg·K. Thus, $h_{ts} = 2611.4$ kJ/kg.

The actual enthalpy of steam at the throat is

$$\eta_N = \frac{h_{01} - h_t}{h_{01} - h_{ts}} \rightarrow 0.90 = \frac{2713.4 - h_t}{2713.4 - 2611.4} \rightarrow h_t = 2621.6 \text{ kJ/kg}$$

Therefore at the throat, $P_2 = 1.728$ MPa and $h_t = 2621.6$ kJ/kg. Thus, $\nu_t = 0.1046$ m³/kg.

Then the throat velocity is determined from the steady-flow energy balance,

$$h_1 + \frac{V_1^2}{2} = h_t + \frac{V_t^2}{2} \rightarrow 0 = h_t - h_1 + \frac{V_t^2}{2}$$

Solving for V_t ,

$$V_t = \sqrt{2(h_1 - h_t)} = \sqrt{2(2713.4 - 2621.6) \text{ kJ/kg} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 428.5 \text{ m/s}$$

Thus the throat area is

$$A_t = \frac{\dot{m} \nu_t}{V_t} = \frac{(6.02 \text{ kg/s})(0.1046 \text{ m}^3/\text{kg})}{(428.5 \text{ m/s})} = 14.70 \times 10^{-4} \text{ m}^2 = \mathbf{14.70 \text{ cm}^2}$$

Fundamentals of Engineering (FE) Exam Problems

17-151 An aircraft is cruising in still air at 5°C at a velocity of 400 m/s. The air temperature at the nose of the aircraft where stagnation occurs is

- (a) 5°C (b) 25°C (c) 55°C (d) 80°C (e) 85°C

Answer (e) 85°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
T1=5 "C"
Vel1= 400 "m/s"
T1_stag=T1+Vel1^2/(2*Cp*1000)
```

"Some Wrong Solutions with Common Mistakes:"

```
W1_Tstag=T1 "Assuming temperature rise"
W2_Tstag=Vel1^2/(2*Cp*1000) "Using just the dynamic temperature"
W3_Tstag=T1+Vel1^2/(Cp*1000) "Not using the factor 2"
```

17-152 Air is flowing in a wind tunnel at 15°C, 80 kPa, and 200 m/s. The stagnation pressure at a probe inserted into the flow stream is

- (a) 82 kPa (b) 91 kPa (c) 96 kPa (d) 101 kPa (e) 114 kPa

Answer (d) 101 kPa

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
T1=15 "K"
P1=80 "kPa"
Vel1= 200 "m/s"
T1_stag=(T1+273)+Vel1^2/(2*Cp*1000) "C"
T1_stag/(T1+273)=(P1_stag/P1)^((k-1)/k)
```

"Some Wrong Solutions with Common Mistakes:"

```
T11_stag/T1=(W1_P1stag/P1)^((k-1)/k); T11_stag=T1+Vel1^2/(2*Cp*1000) "Using deg. C for
temperatures"
T12_stag/(T1+273)=(W2_P1stag/P1)^((k-1)/k); T12_stag=(T1+273)+Vel1^2/(Cp*1000) "Not
using the factor 2"
T13_stag/(T1+273)=(W3_P1stag/P1)^((k-1)/k); T13_stag=(T1+273)+Vel1^2/(2*Cp*1000) "Using
wrong isentropic relation"
```

17-153 An aircraft is reported to be cruising in still air at -20°C and 40 kPa at a Mach number of 0.86. The velocity of the aircraft is

- (a) 91 m/s (b) 220 m/s (c) 186 m/s (d) 280 m/s (e) 378 m/s

Answer (d) 280 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
T1=-10+273 "K"
P1=40 "kPa"
Mach=0.86
VS1=SQRT(k*R*T1*1000)
Mach=Vel1/VS1
```

"Some Wrong Solutions with Common Mistakes:"

```
W1_vel=Mach*VS2; VS2=SQRT(k*R*T1) "Not using the factor 1000"
W2_vel=VS1/Mach "Using Mach number relation backwards"
W3_vel=Mach*VS3; VS3=k*R*T1 "Using wrong relation"
```

17-154 Air is flowing in a wind tunnel at 12°C and 66 kPa at a velocity of 230 m/s. The Mach number of the flow is (Problem changed, 2/2001)

- (a) 0.54 (b) 0.87 (c) 3.3 (d) 0.36 (e) 0.68

Answer (e) 0.68

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
T1=12+273 "K"
P1=66 "kPa"
Vel1=230 "m/s"
VS1=SQRT(k*R*T1*1000)
Mach=Vel1/VS1
```

"Some Wrong Solutions with Common Mistakes:"

```
W1_Mach=Vel1/VS2; VS2=SQRT(k*R*(T1-273)*1000) "Using C for temperature"
W2_Mach=VS1/Vel1 "Using Mach number relation backwards"
W3_Mach=Vel1/VS3; VS3=k*R*T1 "Using wrong relation"
```

17-155 Consider a converging nozzle with a low velocity at the inlet and sonic velocity at the exit plane. Now the nozzle exit diameter is reduced by half while the nozzle inlet temperature and pressure are maintained the same. The nozzle exit velocity will

- (a) remain the same. (b) double. (c) quadruple. (d) go down by half. (e) go down to one-fourth.

Answer (a) remain the same.

17-156 Air is approaching a converging-diverging nozzle with a low velocity at 20°C and 300 kPa, and it leaves the nozzle at a supersonic velocity. The velocity of air at the throat of the nozzle is

- (a) 290 m/s (b) 98 m/s (c) 313 m/s (d) 343 m/s (e) 412 m/s

Answer (c) 313 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
"Properties at the inlet"
T1=20+273 "K"
P1=300 "kPa"
Vel1=0 "m/s"
To=T1 "since velocity is zero"
Po=P1
"Throat properties"
T_throat=2*To/(k+1)
P_throat=Po*(2/(k+1))^(k/(k-1))
"The velocity at the throat is the velocity of sound,"
V_throat=SQRT(k*R*T_throat*1000)
```

"Some Wrong Solutions with Common Mistakes:"

```
W1_Vthroat=SQRT(k*R*T1*1000) "Using T1 for temperature"
W2_Vthroat=SQRT(k*R*T2_throat*1000); T2_throat=2*(To-273)/(k+1) "Using C for temperature"
W3_Vthroat=k*R*T_throat "Using wrong relation"
```

17-157 Argon gas is approaching a converging-diverging nozzle with a low velocity at 20°C and 120 kPa, and it leaves the nozzle at a supersonic velocity. If the cross-sectional area of the throat is 0.015 m², the mass flow rate of argon through the nozzle is

- (a) 0.41 kg/s (b) 3.4 kg/s (c) 5.3 kg/s (d) 17 kg/s (e) 22 kg/s

Answer (c) 5.3 kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.667
Cp=0.5203 "kJ/kg.K"
```

```

R=0.2081 "kJ/kg.K"
A=0.015 "m^2"
"Properties at the inlet"
T1=20+273 "K"
P1=120 "kPa"
Vel1=0 "m/s"
To=T1 "since velocity is zero"
Po=P1
"Throat properties"
T_throat=2*To/(k+1)
P_throat=Po*(2/(k+1))^(k/(k-1))
rho_throat=P_throat/(R*T_throat)
"The velocity at the throat is the velocity of sound,"
V_throat=SQRT(k*R*T_throat*1000)
m=rho_throat*A*V_throat

```

"Some Wrong Solutions with Common Mistakes:"

```

W1_mass=rho_throat*A*V1_throat; V1_throat=SQRT(k*R*T1_throat*1000); T1_throat=2*(To-273)/(k+1) "Using C for temp"
W2_mass=rho2_throat*A*V_throat; rho2_throat=P1/(R*T1) "Using density at inlet"

```

17-158 Carbon dioxide enters a converging-diverging nozzle at 60 m/s, 310°C, and 300 kPa, and it leaves the nozzle at a supersonic velocity. The velocity of carbon dioxide at the throat of the nozzle is

- (a) 125 m/s (b) 225 m/s (c) 312 m/s (d) 353 m/s (e) 377 m/s

Answer (d) 353 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```

k=1.289
Cp=0.846 "kJ/kg.K"
R=0.1889 "kJ/kg.K"
"Properties at the inlet"
T1=310+273 "K"
P1=300 "kPa"
Vel1=60 "m/s"
To=T1+Vel1^2/(2*Cp*1000)
To/T1=(Po/P1)^((k-1)/k)
"Throat properties"
T_throat=2*To/(k+1)
P_throat=Po*(2/(k+1))^(k/(k-1))
"The velocity at the throat is the velocity of sound,"
V_throat=SQRT(k*R*T_throat*1000)

```

"Some Wrong Solutions with Common Mistakes:"

```

W1_Vthroat=SQRT(k*R*T1*1000) "Using T1 for temperature"
W2_Vthroat=SQRT(k*R*T2_throat*1000); T2_throat=2*(T_throat-273)/(k+1) "Using C for temperature"
W3_Vthroat=k*R*T_throat "Using wrong relation"

```

17-159 Consider gas flow through a converging-diverging nozzle. Of the five statements below, select the one that is incorrect:

- (a) The fluid velocity at the throat can never exceed the speed of sound.
- (b) If the fluid velocity at the throat is below the speed of sound, the diversion section will act like a diffuser.
- (c) If the fluid enters the diverging section with a Mach number greater than one, the flow at the nozzle exit will be supersonic.
- (d) There will be no flow through the nozzle if the back pressure equals the stagnation pressure.
- (e) The fluid velocity decreases, the entropy increases, and stagnation enthalpy remains constant during flow through a normal shock.

Answer (c) If the fluid enters the diverging section with a Mach number greater than one, the flow at the nozzle exit will be supersonic.

17-160 Combustion gases with $k = 1.33$ enter a converging nozzle at stagnation temperature and pressure of 400°C and 800 kPa, and are discharged into the atmospheric air at 20°C and 100 kPa. The lowest pressure that will occur within the nozzle is

- (a) 26 kPa
- (b) 100 kPa
- (c) 321 kPa
- (d) 432 kPa
- (e) 272 kPa

Answer (d) 432 kPa

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.33
Po=800 "kPa"
"The critical pressure is"
P_throat=Po*(2/(k+1))^(k/(k-1))
"The lowest pressure that will occur in the nozzle is the higher of the critical or atmospheric pressure."
```

```
"Some Wrong Solutions with Common Mistakes:"
W2_Pthroat=Po*(1/(k+1))^(k/(k-1)) "Using wrong relation"
W3_Pthroat=100 "Assuming atmospheric pressure"
```

17-161 ... 17-163 Design and Essay Problems

