The enthalpy of nitrogen at 400 R and 2000 psia is to be determined using data from the ideal-gas nitrogen table and the generalized enthalpy departure chart.

**Analysis (a)** From the ideal gas table of nitrogen (Table A-18E) we read

\[ h = 2777.0 \text{ Btu/lbmol} = 99.18 \text{ Btu/lbm} \quad (M_{N_2} = 28 \text{ lbm/lbmol}) \]

at the specified temperature. This value involves 44.2% error.

(b) The enthalpy departure of nitrogen at the specified state is determined from the generalized chart to be (Fig. A-29)

\[
\begin{align*}
T_R &= \frac{T}{T_{cr}} = \frac{400}{227.1} = 1.761 \\
P_R &= \frac{P}{P_{cr}} = \frac{2000}{492} = 4.065 \\
\therefore \quad Z_h &= \left( \frac{\left( h_{\text{ideal}} - \bar{h} \right)_{T,P}}{R_u T_{cr}} \right) = 1.18
\end{align*}
\]

Thus,

\[
\bar{h} = \bar{h}_{\text{ideal}} - Z_h R_u T_{cr} = 2777.0 - \left[ (1.18)(1.986)(227.1) \right] = 2244.8 \text{ Btu/lbmol}
\]

or,

\[
\frac{h}{M} = \frac{2244.8}{28 \text{ lbm/lbmol}} = 80.17 \text{ Btu/lbm} \quad (54.9\% \text{ error})
\]

The errors involved in the enthalpy and internal energy of CO$_2$ at 350 K and 10 MPa if it is assumed to be an ideal gas are to be determined.

**Analysis (a)** The enthalpy departure of CO$_2$ at the specified state is determined from the generalized chart to be (Fig. A-29)

\[
\begin{align*}
T_R &= \frac{T}{T_{cr}} = \frac{350}{304.2} = 1.151 \\
P_R &= \frac{P}{P_{cr}} = \frac{10}{7.39} = 1.353 \\
\therefore \quad Z_h &= \left( \frac{\left( h_{\text{ideal}} - \bar{h} \right)_{T,P}}{R_u T_{cr}} \right) = 1.5
\end{align*}
\]

Thus,

\[
\bar{h} = \bar{h}_{\text{ideal}} - Z_h R_u T_{cr} = 11,351 - \left[ (1.5)(8.314)(304.2) \right] = 7,557 \text{ kJ/kmol}
\]

and,

\[
\text{Error} = \left( \frac{\left( h_{\text{ideal}} - \bar{h} \right)_{T,P}}{\bar{h}} \right) = \frac{11,351 - 7,557}{7,557} = 50.2\%
\]

(b) At the calculated $T_R$ and $P_R$ the compressibility factor is determined from the compressibility chart to be $Z = 0.65$. Then using the definition of enthalpy, the internal energy is determined to be

\[
\bar{u} = \bar{h} - P\bar{v} = \bar{h} - ZR_u T = 7557 - \left[ (0.65)(8.314)(350) \right] = 5,666 \text{ kJ/kmol}
\]

and,

\[
\text{Error} = \left( \frac{\left( u_{\text{ideal}} - \bar{u} \right)}{\bar{u}} \right) = \frac{8,439 - 5,666}{5,666} = 48.9\%
\]
The enthalpy and entropy changes of nitrogen during a process are to be determined assuming ideal gas behavior and using generalized charts.

**Analysis (a)** Using data from the ideal gas property table of nitrogen (Table A-18),

\[
(h_2 - h_1)_{\text{ideal}} = h_{2, \text{ideal}} - h_{1, \text{ideal}} = 9306 - 6,537 = 2769 \text{ kJ/kmol}
\]

and

\[
(s_2 - s_1)_{\text{ideal}} = s^*_{2} - s^*_{1} - R_u \ln \frac{P_2}{P_1} = 193.562 - 183.289 - 8.314 \times \ln \frac{12}{6} = 4.510 \text{ kJ/kmol} \cdot \text{K}
\]

(b) The enthalpy and entropy departures of nitrogen at the specified states are determined from the generalized charts to be (Figs. A-29, A-30)

\[
\begin{align*}
T_{R1} &= \frac{T_1}{T_{cr}} = \frac{225}{126.2} = 1.783 \\
P_{R1} &= \frac{P_1}{P_{cr}} = \frac{6}{3.39} = 1.770 \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
\end{align*}
\]

\[
\begin{align*}
Z_{hl} &= 0.6 \text{ and } Z_{s1} = 0.25 \\
Z_{h2} &= 0.4 \text{ and } Z_{s2} = 0.15
\end{align*}
\]

Substituting,

\[
\begin{align*}
\bar{h}_2 - \bar{h}_1 &= R_u T_{cr} (Z_{hl} - Z_{h2}) + (\bar{h}_2 - \bar{h}_1)_{\text{ideal}} \\
&= (8.314)(126.2)(0.6 - 0.4) + 2769 = 2979 \text{ kJ/kmol}
\end{align*}
\]

\[
\begin{align*}
\bar{s}_2 - \bar{s}_1 &= R_u (Z_{s1} - Z_{s2}) + (\bar{s}_2 - \bar{s}_1)_{\text{ideal}} \\
&= (8.314)(0.25 - 0.15) + 4.510 = 5.341 \text{ kJ/kmol} \cdot \text{K}
\end{align*}
\]
12-61 The enthalpy and entropy changes of CO$_2$ during a process are to be determined assuming ideal gas behavior and using generalized charts.

**Analysis (a)** Using data from the ideal gas property table of CO$_2$ (Table A-20),

\[
(h_2 - h_1)_{\text{ideal}} = \frac{\overline{h}_2 - \overline{h}_1}{M} = \frac{8697 - 7627}{12} = 1070 \text{ kJ/kg mol}
\]

\[
(s_2 - s_1)_{\text{ideal}} = s'_2 - s'_1 - R_u = 211.376 - 207.337 - 8.314 \times \ln \frac{12}{7} = -0.442 \text{ kJ/kg m mol K}
\]

Thus,

\[
(h_2 - h_1)_{\text{ideal}} = \frac{\overline{h}_2 - \overline{h}_1}{M} = \frac{1070 \text{ kJ/kg m mol}}{44 \text{ kg/mol}} = 24.32 \text{ kJ/kg}
\]

\[
(s_2 - s_1)_{\text{ideal}} = \frac{\overline{s}_2 - \overline{s}_1}{M} = \frac{-0.442 \text{ kJ/kg m mol}}{44 \text{ kg/mol}} = -0.0100 \text{ kJ/kg K}
\]

(b) The enthalpy and entropy departures of CO$_2$ at the specified states are determined from the generalized charts to be (Figs. A-29, A-30)

\[
T_{R1} = \frac{T_1}{T_{cr}} = \frac{250}{304.2} = 0.822 \quad \rightarrow Z_{h1} = 5.5 \text{ and } Z_{s1} = 5.3
\]

\[
P_{R1} = \frac{P_1}{P_{cr}} = \frac{7}{7.39} = 0.947
\]

and

\[
T_{R2} = \frac{T_2}{T_{cr}} = \frac{280}{304.2} = 0.920 \quad \rightarrow Z_{h2} = 5.0 \text{ and } Z_{s2} = 4.2
\]

\[
P_{R2} = \frac{P_2}{P_{cr}} = \frac{12}{7.39} = 1.624
\]

Thus,

\[
h_2 - h_1 = RT_{cr}(Z_{h1} - Z_{h2}) + (h_2 - h_1)_{\text{ideal}} = (0.1889)(304.2)(5.5 - 5.0) + 24.32 = 53.05 \text{ kJ/kg}
\]

\[
s_2 - s_1 = R(Z_{s1} - Z_{s2}) + (s_2 - s_1)_{\text{ideal}} = (0.1889)(5.3 - 4.2) - 0.010 = 0.198 \text{ kJ/kg K}
\]
Methane is compressed adiabatically by a steady-flow compressor. The required power input to the compressor is to be determined using the generalized charts.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

The steady-flow energy balance equation for this compressor can be expressed as

\[ \dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta \dot{E}_{\text{system}} \]  
\[ \dot{E}_{\text{in}} = \dot{E}_{\text{out}} \]  
\[ \dot{W}_{C,\text{in}} + \dot{m}h_1 = \dot{m}h_2 \]  
\[ \dot{W}_{C,\text{in}} = \dot{m}(h_2 - h_1) \]

The enthalpy departures of CH₄ at the specified states are determined from the generalized charts to be (Fig. A-29)

\[ T_{R1} = \frac{T_1}{T_{cr}} = \frac{263}{191.1} = 1.376 \]  
\[ P_{R1} = \frac{P_1}{P_{cr}} = \frac{2}{4.64} = 0.431 \]  
\[ \rightarrow Z_{h1} = 0.21 \]

and

\[ T_{R2} = \frac{T_2}{T_{cr}} = \frac{383}{191.1} = 2.00 \]  
\[ P_{R2} = \frac{P_2}{P_{cr}} = \frac{10}{4.64} = 2.155 \]  
\[ \rightarrow Z_{h2} = 0.50 \]

Thus,

\[ h_2 - h_1 = RT_{cr}(Z_{h1} - Z_{h2}) + (h_2 - h_1)_{\text{ideal}} \]  
\[ = (0.5182)(191.1)(0.21 - 0.50) + 2.2537(110 - (-10)) = 241.7 \text{ kJ/kg} \]

Substituting,

\[ \dot{W}_{C,\text{in}} = (0.55 \text{ kg/s})(241.7 \text{ kJ/kg}) = 133 \text{ kW} \]
Propane is compressed isothermally by a piston-cylinder device. The work done and the heat transfer are to be determined using the generalized charts.

**Assumptions**
1. The compression process is quasi-equilibrium.
2. Kinetic and potential energy changes are negligible.

**Analysis**
(a) The enthalpy departure and the compressibility factors of propane at the initial and the final states are determined from the generalized charts to be (Figs. A-29, A-15)

\[
\begin{align*}
T_{R1} &= \frac{T_1}{T_{cr}} = \frac{373}{370} = 1.008 \quad \rightarrow Z_{h1} = 0.28 \text{ and } Z_1 = 0.92 \\
P_{R1} &= \frac{P_1}{P_{cr}} = \frac{1}{4.26} = 0.235
\end{align*}
\]

and

\[
\begin{align*}
T_{R2} &= \frac{T_2}{T_{cr}} = \frac{373}{370} = 1.008 \quad \rightarrow Z_{h2} = 1.8 \text{ and } Z_2 = 0.50 \\
P_{R2} &= \frac{P_2}{P_{cr}} = \frac{4}{4.26} = 0.939
\end{align*}
\]

Treating propane as a real gas with \(Z_{avg} = \frac{Z_1 + Z_2}{2} = \frac{(0.92 + 0.50)}{2} = 0.71\),

\[P \nu = ZRT \equiv Z_{avg}RT = C = \text{constant}\]

Then the boundary work becomes

\[
w_{b,\text{in}} = -\int P \, d\nu = -\int \frac{C}{\nu} \, d\nu = -C \ln \frac{\nu_2}{\nu_1} = Z_{avg}RT \ln \frac{Z_2RT / P_2}{Z_1RT / P_1} = -Z_{ave}RT \ln \frac{Z_2P_1}{Z_1P_2} = -(0.71)(0.1885 \text{ kJ/kg} \cdot \text{K})(373 \text{ K}) \ln \frac{(0.50)(1)}{(0.92)(4)} = 99.6 \text{ kJ/kg}
\]

Also,

\[
h_2 - h_1 = RT_{cr}(Z_{h1} - Z_{h2}) + (h_2 - h_1)_{\text{ideal}} = (0.1885)(370)(0.28 - 1.8) + 0 = -106 \text{ kJ/kg}
\]

\[
u_2 - u_1 = (h_2 - h_1) - R(Z_2T_2 - Z_1T_1) = -106 - (0.1885)(0.5)(373) - (0.92)(373) = -76.5 \text{ kJ/kg}
\]

Then the heat transfer for this process is determined from the closed system energy balance to be

\[
E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}
\]

\[
q_{\text{in}} + w_{b,\text{in}} = \Delta u = u_2 - u_1
\]

\[
q_{\text{in}} = (u_2 - u_1) - w_{b,\text{in}} = -76.5 - 99.6 = -176.1 \text{ kJ/kg} \quad \rightarrow \quad q_{\text{out}} = 176.1 \text{ kJ/kg}
\]
Problem 12-63 is reconsidered. This problem is to be extended to compare the solutions based on the ideal gas assumption, generalized chart data and real fluid (EES) data. Also, the solution is to be extended to carbon dioxide, nitrogen and methane.

**Analysis** The problem is solved using EES, and the solution is given below.

```plaintext
Procedure INFO(Name$, T[1] : Fluid$, T_critical, p_critical)
    If Name$='Propane' then
        T_critical=370 ; p_critical=4620 ; Fluid$='C3H8'; goto 10
    endif
    If Name$='Methane' then
        T_critical=191.1 ; p_critical=4640 ; Fluid$='CH4'; goto 10
    endif
    If Name$='Nitrogen' then
        T_critical=126.2 ; p_critical=3390 ; Fluid$='N2'; goto 10
    endif
    If Name$='Oxygen' then
        T_critical=154.8 ; p_critical=5080 ; Fluid$='O2'; goto 10
    endif
    If Name$='CarbonDioxide' then
        T_critical=304.2 ; p_critical=7390 ; Fluid$='CO2'; goto 10
    endif
    If Name$='n-Butane' then
        T_critical=425.2 ; p_critical=3800 ; Fluid$='C4H10'; goto 10
    endif
10:
    If T[1]<=T_critical then
        CALL ERROR('The supplied temperature must be greater than the critical temperature for the fluid. A value of XXXF1 K was supplied',T[1])
    endif
end

{"Data from the Diagram Window"
T[1]=100+273.15
p[1]=1000
p[2]=4000
Name$='Propane'
Fluid$='C3H8'
}

Call INFO(Name$, T[1] : Fluid$, T_critical, p_critical)

R_u=8.314
M=molarmass(Fluid$)
R=R_u/M

"****** IDEAL GAS SOLUTION ******
"State 1"
h_ideal[1]=enthalpy(Fluid$, T=T[1])  "Enthalpy of ideal gas"
s_ideal[1]=entropy(Fluid$, T=T[1], p=p[1])  "Entropy of ideal gas"
"State 2"
h_ideal[2]=enthalpy(Fluid$, T=T[2])  "Enthalpy of ideal gas"
s_ideal[2]=entropy(Fluid$, T=T[2], p=p[2])  "Entropy of ideal gas"

****** IDEAL GAS SOLUTION ******
"State 1"
h_ideal[1]=enthalpy(Fluid$, T=T[1])  "Enthalpy of ideal gas"
s_ideal[1]=entropy(Fluid$, T=T[1], p=p[1])  "Entropy of ideal gas"
"State 2"
h_ideal[2]=enthalpy(Fluid$, T=T[2])  "Enthalpy of ideal gas"
s_ideal[2]=entropy(Fluid$, T=T[2], p=p[2])  "Entropy of ideal gas"
```

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"Work is the integral of p dv, which can be done analytically."

\[ w_{\text{ideal}} = R^*T[1]^*\ln(p[1]/p[2]) \]

"First Law - note that \( u_{\text{ideal}}[2] \) is equal to \( u_{\text{ideal}}[1] \)

\[ q_{\text{ideal}} - w_{\text{ideal}} = u_{\text{ideal}}[2] - u_{\text{ideal}}[1] \]

"Entropy change"

\[ \Delta s_{\text{ideal}} = s_{\text{ideal}}[2] - s_{\text{ideal}}[1] \]

"***** COMPRESSIBILITY CHART SOLUTION *****"

"State 1"

\[ T[1] = T[1]/T_{\text{critical}} \]
\[ p[1] = p[1]/p_{\text{critical}} \]
\[ Z[1] = \text{COMPRESS}(T[1], p[1]) \]
\[ \Delta H[1] = \text{ENTHDEP}(T[1], p[1])^*R^*T_{\text{critical}} \]
\[ h[1] = h_{\text{ideal}}[1] - \Delta H[1] \]

"Internal energy of gas using charts"

\[ \Delta s[1] = \text{ENTRDEP}(T[1], p[1])^*R \]
\[ s[1] = s_{\text{ideal}}[1] - \Delta s[1] \]

"Entropy of real gas using charts"

"State 2"

\[ p[2] = p[2]/p_{\text{critical}} \]
\[ Z[2] = \text{COMPRESS}(T[2], p[2]) \]
\[ \Delta H[2] = \text{ENTHDEP}(T[2], p[2])^*R^*T_{\text{critical}} \]

"Internal energy of gas using charts"

"Work using charts - note use of EES integral function to evaluate the integral of p dv."

\[ w_{\text{chart}} = \text{Integral}(p, v, v[1], v[2]) \]

"We need an equation to relate \( p \) and \( v \) in the above INTEGRAL function."

\[ p^*v = \text{COMPRESS}(T[2], p/p_{\text{critical}})^*R^*T[1] \]

"To specify relationship between \( p \) and \( v \)"

"Find the limits of integration"

\[ "to get v[1], the lower bound" \]
\[ "to get v[2], the upper bound" \]

"First Law - note that \( u[2] \) is not equal to \( u[1] \)

\[ q_{\text{chart}} - w_{\text{chart}} = u[2] - u[1] \]

"Entropy Change"

\[ \Delta s_{\text{chart}} = s[2] - s[1] \]

"***** SOLUTION USING EES BUILT-IN PROPERTY DATA *****"

"At state 1"

\[ u_{\text{ees}}[1] = \text{intEnergy}(\text{Name$}, T = T[1], p = p[1]) \]
\[ s_{\text{ees}}[1] = \text{entropy}(\text{Name$}, T = T[1], p = p[1]) \]

"At state 2"

\[ u_{\text{ees}}[2] = \text{intEnergy}(\text{Name$}, T = T[2], p = p[2]) \]
\[ s_{\text{ees}}[2] = \text{entropy}(\text{Name$}, T = T[2], p = p[2]) \]

"Work using EES built-in properties - note use of EES Integral function to evaluate the integral of pdv."

\[ w_{\text{ees}} = \text{integral}(p_{\text{ees}}, v_{\text{ees}}, v_{\text{ees}}[1], v_{\text{ees}}[2]) \]

"The following equation relates \( p \) and \( v \) in the above INTEGRAL"
p_ees=pressure(Name$, T=T[1], v=v_ees) "To specify relationship between p and v"

"Find the limits of integration"

v_ees[1]=volume(Name$, T=T[1], p=p[1]) "to get lower bound"
v_ees[2]=volume(Name$, T=T[2], p=p[2]) "to get upper bound"

"First law - note that u_ees[2] is not equal to u_ees[1]"
q_ees-w_ees=u_ees[2]-u_ees[1]

"Entropy change"
DELTAs_ees=s_ees[2]-s_ees[1]

"Note: In all three solutions to this problem we could have calculated the heat transfer by
q/T=DELTAs since T is constant. Then the first law could have been used to find the work.
The use of integral of p dv to find the work is a more fundamental approach and can be
used if T is not constant."

SOLUTION

<table>
<thead>
<tr>
<th>Fluid$='C3H8'</th>
</tr>
</thead>
<tbody>
<tr>
<td>DELTAh[1]=16.48 [kJ/kg]</td>
</tr>
<tr>
<td>DELTAh[2]=91.96 [kJ/kg]</td>
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<tr>
<td>DELTAs[1]=0.03029 [kJ/kg-K]</td>
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<tr>
<td>DELTAs[2]=0.1851 [kJ/kg-K]</td>
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<tr>
<td>DELTAs_chart=-0.4162 [kJ/kg-K]</td>
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<tr>
<td>DELTAs_ees=-0.4711 [kJ/kg-K]</td>
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<tr>
<td>DELTAs_ideal=-0.2614 [kJ/kg-K]</td>
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<tr>
<td>h[1]=2232 [kJ/kg]</td>
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<tr>
<td>h[2]=2308 [kJ/kg]</td>
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<tr>
<td>h_ideal[1]=2216 [kJ/kg]</td>
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<tr>
<td>h_ideal[2]=2216 [kJ/kg]</td>
</tr>
<tr>
<td>M=44.1</td>
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<tr>
<td>Name$='Propane'</td>
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<td>p=4000</td>
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<tr>
<td>p[2]=4000 [kPa]</td>
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<td>pr[2]=0.8658</td>
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<td>q_chart=-155.3 [kJ/kg]</td>
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<td>q_ees=-175.8 [kJ/kg]</td>
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<tr>
<td>q_ideal=-97.54 [kJ/kg]</td>
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<tr>
<td>R=0.1885 [kJ/kg-K]</td>
</tr>
<tr>
<td>R_u=8.314 [kJ/mole-K]</td>
</tr>
<tr>
<td>s[1]=6.073 [kJ/kg-K]</td>
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<tr>
<td>s[2]=5.657 [kJ/kg-K]</td>
</tr>
<tr>
<td>s_ees[1]=2.797 [kJ/kg-K]</td>
</tr>
<tr>
<td>s_ees[2]=2.326 [kJ/kg-K]</td>
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<tr>
<td>s_ideal[1]=6.103 [kJ/kg-K]</td>
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<tr>
<td>s_ideal[2]=5.842 [kJ/kg-K]</td>
</tr>
<tr>
<td>T[1]=373.2 [K]</td>
</tr>
<tr>
<td>T[2]=373.2 [K]</td>
</tr>
<tr>
<td>Tr[1]=1.009</td>
</tr>
<tr>
<td>Tr[2]=1.009</td>
</tr>
<tr>
<td>T_critical=370 [K]</td>
</tr>
<tr>
<td>u[1]=-2298 [kJ/kg]</td>
</tr>
<tr>
<td>u[2]=-2351 [kJ/kg]</td>
</tr>
<tr>
<td>u_ees[1]=688.4 [kJ/kg]</td>
</tr>
<tr>
<td>u_ees[2]=617.1 [kJ/kg]</td>
</tr>
<tr>
<td>u_ideal[1]=-2286 [kJ/kg]</td>
</tr>
<tr>
<td>u_ideal[2]=-2286 [kJ/kg]</td>
</tr>
<tr>
<td>v=0.01074</td>
</tr>
<tr>
<td>v[1]=0.06506 [m^3/kg]</td>
</tr>
<tr>
<td>v[2]=0.01074 [m^3/kg]</td>
</tr>
<tr>
<td>v_ees=0.009426</td>
</tr>
<tr>
<td>v_ees[1]=0.0646 [m^3/kg]</td>
</tr>
<tr>
<td>v_ees[2]=0.009426 [m^3/kg]</td>
</tr>
<tr>
<td>w_chart=-101.9 [kJ/kg]</td>
</tr>
<tr>
<td>w_chart=-104.5 [kJ/kg]</td>
</tr>
<tr>
<td>w_ees=-97.54 [kJ/kg]</td>
</tr>
<tr>
<td>w_ideal=-97.54 [kJ/kg]</td>
</tr>
<tr>
<td>Z[1]=0.9246</td>
</tr>
<tr>
<td>Z[2]=0.6104</td>
</tr>
</tbody>
</table>
Propane is compressed isothermally by a piston-cylinder device. The work done and the heat transfer are to be determined using the generalized charts.

**Assumptions**

1. The compression process is quasi-equilibrium.
2. Kinetic and potential energy changes are negligible.
3. The device is well-insulated and thus heat transfer is negligible.

**Analysis (a)**

The enthalpy departure and the compressibility factors of propane at the initial and the final states are determined from the generalized charts to be (Figs. A-29, A-15)

\[
\begin{align*}
T_{R1} &= \frac{T_i}{T_{cr}} = \frac{660}{665.9} = 0.991 \\
P_{R1} &= \frac{P_i}{P_{cr}} = \frac{200}{617} = 0.324
\end{align*}
\]

\[\rightarrow Z_{h1} = 0.37 \text{ and } Z_1 = 0.88\]

\[
\begin{align*}
T_{R2} &= \frac{T_f}{T_{cr}} = \frac{660}{665.9} = 0.991 \\
P_{R2} &= \frac{P_f}{P_{cr}} = \frac{800}{617} = 1.297
\end{align*}
\]

\[\rightarrow Z_{h2} = 4.2 \text{ and } Z_2 = 0.22\]

Treating propane as a real gas with \(Z_{avg} = \frac{Z_1 + Z_2}{2} = \frac{0.88 + 0.22}{2} = 0.55,\)

\[P \nu = ZRT \equiv Z_{avg}RT = C = \text{constant}\]

Then the boundary work becomes

\[w_{h,in} = -\int P d\nu = -\int \frac{C}{\nu} d\nu = -C \ln \frac{\nu_f}{\nu_i} = -Z_{avg}RT \ln \frac{\frac{Z_2}{Z_1}RT/P_2}{\frac{Z_2}{Z_1}RT/P_1} = -Z_{avg}RT \ln \frac{Z_2P_1}{Z_1P_2}\]

\[= -(0.55)(0.04504 \text{ Btu/lbm} \cdot \text{R})(660 \text{ R}) \ln \left(\frac{0.22}{0.88}\right) = 45.3 \text{ Btu/lbm}\]

Also,

\[h_2 - h_1 = RT_{cr}(Z_{h1} - Z_{h2}) + (h_2 - h_1)_{ideal}\]

\[= (0.04504)(665.9)(0.37 - 4.2) + 0 = -114.9 \text{ Btu/lbm}\]

\[u_2 - u_1 = (h_2 - h_1) - R(Z_2T_2 - Z_1T_1)\]

\[= -114.9 - (0.04504)((0.22)(660) - (0.88)(660)) = -95.3 \text{ Btu/lbm}\]

Then the heat transfer for this process is determined from the closed system energy balance equation to be

\[E_{in} - E_{out} = \Delta E_{system}\]

\[q_{in} + w_{h,in} = \Delta u = u_2 - u_1\]

\[q_{in} = (u_2 - u_1) - w_{h,in} = -95.3 - 45.3 = -140.6 \text{ Btu/lbm} \rightarrow q_{out} = 140.6 \text{ Btu/lbm}\]
Propane is compressed isothermally by a piston-cylinder device. The exergy destruction associated with this process is to be determined.

**Assumptions**

1. The compression process is quasi-equilibrium.
2. Kinetic and potential energy changes are negligible.

**Properties**
The gas constant of propane is \( R = 0.1885 \text{ kJ/kg} \cdot \text{K} \) (Table A-1).

**Analysis**
The exergy destruction is determined from its definition \( x_{\text{destroyed}} = T_0 s_{\text{gen}} \) where the entropy generation is determined from an entropy balance on the contents of the cylinder. It gives

\[
S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}} = -\frac{Q_{\text{out}}}{T_{b,\text{surr}}} + S_{\text{gen}} = m(s_2 - s_1) \rightarrow s_{\text{gen}} = (s_2 - s_1) + \frac{Q_{\text{out}}}{T_{\text{surr}}}
\]

where

\[
\Delta S_{\text{sys}} = s_2 - s_1 = R(Z_{s1} - Z_{s2}) + (s_2 - s_1)_{\text{ideal}}
\]

\[
(s_2 - s_1)_{\text{ideal}} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 0 - 0.1885 \ln \frac{4}{1} = -0.261 \text{ kJ/kg} \cdot \text{K}
\]

\[
T_{R1} = \frac{T_1}{T_{cr}} = \frac{373}{370} = 1.008 \quad \rightarrow \quad Z_{s1} = 0.21
\]

\[
P_{R1} = \frac{P_1}{P_{cr}} = \frac{1}{4.26} = 0.235
\]

and

\[
T_{R2} = \frac{T_2}{T_{cr}} = \frac{373}{370} = 1.008 \quad \rightarrow \quad Z_{s2} = 1.5
\]

\[
P_{R2} = \frac{P_2}{P_{cr}} = \frac{4}{4.26} = 0.939
\]

Thus,

\[
\Delta S_{\text{sys}} = s_2 - s_1 = R(Z_{s1} - Z_{s2}) + (s_2 - s_1)_{\text{ideal}} = (0.1885)(0.21 - 1.5) - 0.261 = -0.504 \text{ kJ/kg} \cdot \text{K}
\]

and

\[
x_{\text{destroyed}} = T_0 s_{\text{gen}} = T_0 \left( s_2 - s_1 \right) + \frac{Q_{\text{out}}}{T_{\text{surr}}} = (303 \text{ K}) \left( -0.504 + \frac{176.1 \text{ kJ/kg}}{303 \text{ K}} \right) \text{ kJ/kg} \cdot \text{K} = 23.4 \text{ kJ/kg}
\]
Carbon dioxide passes through an adiabatic nozzle. The exit velocity is to be determined using the generalized enthalpy departure chart.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.
3. The nozzle is adiabatic and thus heat transfer is negligible.

**Properties**
The gas constant of CO\(_2\) is 0.1889 kJ/kg.K (Table A-1).

**Analysis**
The steady-flow energy balance equation for this nozzle can be expressed as

\[
\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system} = 0
\]

\[
\dot{E}_{in} = \dot{E}_{out}
\]

\[
h_1 + (V_1^2 / 2) = h_2 + (V_2^2 / 2)
\]

\[
V_2 = \sqrt{2(h_1 - h_2)}
\]

The enthalpy departures of CO\(_2\) at the specified states are determined from the generalized enthalpy departure chart to be

\[
T_{R1} = \frac{T_1}{T_{cr}} = \frac{450}{304.2} = 1.48 \quad \rightarrow \quad Z_{h1} = 0.55
\]

\[
P_{R1} = \frac{P_1}{P_{cr}} = \frac{8}{7.39} = 1.08
\]

and

\[
T_{R2} = \frac{T_2}{T_{cr}} = \frac{350}{304.2} = 1.15 \quad \rightarrow \quad Z_{h2} = 0.20
\]

\[
P_{R2} = \frac{P_2}{P_{cr}} = \frac{2}{7.39} = 0.27
\]

Thus,

\[
h_2 - h_1 = RT_{cr} (Z_{h1} - Z_{h2}) + (h_2 - h_1)_{\text{ideal}}
\]

\[
= (0.1889)(304.2)(0.55 - 0.2) + (11,351 - 15,483)/44 = -73.8 \text{ kJ/kg}
\]

Substituting,

\[
V_2 = \sqrt{2(73.8 \text{ kJ/kg}) \left( \frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 384 \text{ m/s}
\]
Problem 12-67 is reconsidered. The exit velocity to the nozzle assuming ideal gas behavior, the
generalized chart data, and EES data for carbon dioxide are to be compared.

**Analysis** The problem is solved using EES, and the results are given below.

```plaintext
Procedure INFO(Name$: Fluid$, T_critical, p_critical)
If Name$='CarbonDioxide' then
    T_critical=304.2 ; p_critical=7390 ; Fluid$='CO2'
endif
END

T[1]=450 [K]
P[1]=8000 [kPa]
Name$='CarbonDioxide'
Call INFO(Name$: Fluid$, T_critical, P_critical)
R_u=8.314
M=molarmass(Fluid$)
R=R_u/M

"****** IDEAL GAS SOLUTION ******
"State 1 nd 2"
h_ideal[1]=enthalpy(Fluid$, T=T[1])  "Enthalpy of ideal gas"
h_ideal[2]=enthalpy(Fluid$, T=T[2])  "Enthalpy of ideal gas"
"Exit velocity."
V_2_ideal=SQRT(2*(h_ideal[1]-h_ideal[2])*convert(kJ/kg,m^2/s^2))"[m/s]"

"****** COMPRESSIBILITY CHART SOLUTION ******
"State 1"
Tr[1]=T[1]/T_critical
Pr[1]=P[1]/P_critical
DELTAh[1]=ENTHDEP(Tr[1], Pr[1])*R*T_critical  "Enthalpy departure"

"State 2"
Pr[2]=P[2]/P_critical
DELTAh[2]=ENTHDEP(Tr[2], Pr[2])*R*T_critical  "Enthalpy departure"

"Exit velocity."
V_2_EnthDep=SQRT(2*(h[1]-h[2])*convert(kJ/kg,m^2/s^2))"[m/s]"

"****** SOLUTION USING EES BUILT-IN PROPERTY DATA ******
"At state 1 and 2"
h_ees[1]=enthalpy(Name$, T=T[1], P=P[1])
h_ees[2]=enthalpy(Name$, T=T[2], P=P[2])

"Exit velocity."
V_2_ees=SQRT(2*(h_ees[1]-h_ees[2])*convert(kJ/kg,m^2/s^2))"[m/s]"
```

**PROPRIETARY MATERIAL.** © 2006 The McGraw-Hill Companies, Inc. Limited distribution permitted only to teachers and
educators for course preparation. If you are a student using this Manual, you are using it without permission.
A paddle-wheel placed in a well-insulated rigid tank containing oxygen is turned on. The final pressure in the tank and the paddle-wheel work done during this process are to be determined.

**Assumptions**

1. The tank is well-insulated and thus heat transfer is negligible.
2. Kinetic and potential energy changes are negligible.

**Properties**

The gas constant of O$_2$ is $R = 0.2598$ kJ/kg·K (Table A-1).

**Analysis**

(a) The compressibility factor of oxygen at the initial state is determined from the generalized chart to be

\[
\begin{align*}
T_{R1} &= \frac{T_1}{T_{cr}} = \frac{220}{154.8} = 1.42 \\
\frac{P_{R1}}{P_{cr}} &= \frac{P_1}{P_{cr}} = \frac{10}{5.08} = 1.97
\end{align*}
\]

Then,

\[
P\nu = ZRT \quad \Rightarrow \quad \nu_1 = \frac{(0.8)(0.2598 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(220 \text{ K})}{10,000 \text{ kPa}} = 0.00457 \text{ m}^3/\text{kg}
\]

The specific volume of oxygen remains constant during this process, $\nu_2 = \nu_1$. Thus,

\[
\begin{align*}
T_{R2} &= \frac{T_2}{T_{cr}} = \frac{250}{154.8} = 1.615 \\
\frac{\nu_2}{\nu_1} &= \frac{0.00457 \text{ m}^3/\text{kg}}{17.5 \text{ kg}} = 0.00457 \text{ m}^3/\text{kg}
\end{align*}
\]

\[
\begin{align*}
\nu_2 &= \frac{RT_{cr} / P_{cr}}{(0.2598 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(154.8 \text{ K})/(5080 \text{ kPa})} = 0.577 \\
Z_2 &= \frac{Z_1}{Z_{h1}} = \frac{8.7}{1.15} = 0.757 \\
P_{R2} &= \frac{P_2}{P_{R2}} = (2.4)(5080) = 12,190 \text{ kPa}
\end{align*}
\]

(b) The energy balance relation for this closed system can be expressed as

\[
E_{in} - E_{out} = \Delta E_{system}
\]

\[
W_{in} = \Delta U = m(u_2 - u_1)
\]

\[
W_{in} = m[h_2 - h_1 - (P_2\nu_2 - P_1\nu_1)] = m[h_2 - h_1 - R(Z_2T_2 - Z_1T_1)]
\]

where

\[
h_2 - h_1 = RT_{cr}(Z_1 - Z_{h1}) + (h_2 - h_1)_{ideal}
\]

\[
= (0.2598)(154.8)(1.15 - 1) + (7275 - 6404) / 32 = 33.25 \text{ kJ/kg}
\]

Substituting,

\[
W_{in} = (17.5 \text{ kg})[33.25 - (0.2598 \text{ kJ/kg} \cdot \text{K})(0.87)(250) - (0.80)(220)]\text{K}] = 393 \text{ kJ}
\]
The heat transfer and entropy changes of CO2 during a process are to be determined assuming ideal gas behavior, using generalized charts, and real fluid (EES) data.

**Analysis**

The temperature at the final state is

\[ T_2 = T_1 \frac{P_2}{P_1} = \left( 100 + 273 \right) \frac{8 \text{ MPa}}{1 \text{ MPa}} = 2984 \text{ K} \]

Using data from the ideal gas property table of CO2 (Table A-20),

\[ (\overline{h}_2 - \overline{h}_1)_{\text{ideal}} = 161,293 - 12,269 = 149,024 \text{ kJ/kmol} \]

\[ (\overline{s}_2 - \overline{s}_1)_{\text{ideal}} = s_2' - s_1' - R \ln \frac{P_2}{P_1} = 333.770 - 222.367 - 8.314 \times \ln \frac{8}{1} = 94.115 \text{ kJ/kmol} \cdot \text{K} \]

\[ (h_2 - h_1)_{\text{ideal}} = \frac{(\overline{h}_2 - \overline{h}_1)_{\text{ideal}}}{M} = \frac{149,024 \text{ kJ/kmol}}{44 \text{ kg/kmol}} = 3386.9 \text{ kJ/kg} \]

The heat transfer is determined from an energy balance noting that there is no work interaction

\[ q_{\text{ideal}} = (u_2 - u_1)_{\text{ideal}} = (h_2 - h_1)_{\text{ideal}} - R(T_2 - T_1) \]

\[ = 3386.9 \text{ kJ/kg} - (0.1889 \text{ kJ/kg} \cdot \text{K})(2984 - 373) = 2893.7 \text{ kJ/kg} \]

The entropy change is

\[ \Delta s_{\text{ideal}} = (s_2 - s_1)_{\text{ideal}} = \frac{(\overline{s}_2 - \overline{s}_1)_{\text{ideal}}}{M} = \frac{94.115 \text{ kJ/kmol}}{44 \text{ kg/kmol}} = 2.1390 \text{ kJ/kg} \cdot \text{K} \]

The compressibility factor and the enthalpy and entropy departures of CO2 at the specified states are determined from the generalized charts to be (we used EES)

\[ R_{hl} \]

\[ R_{s1} \]

\[ R_{s2} \]

Thus,

\[ q_{\text{chart}} = u_2 - u_1 = (h_2 - h_1)_{\text{ideal}} - RT_{cr}(Z_{h2} - Z_{hl}) - Z_1(R(T_2 - T_1)) \]

\[ = 3386.9 - (0.1889)(304.2)(-0.1144 - 0.1028) - (0.976)(0.1889)(2887 - 373) = 2935.9 \text{ kJ/kg} \]

\[ \Delta s_{\text{chart}} = (s_2 - s_1)_{\text{chart}} = R(Z_{s1} - Z_{s2}) + (s_2 - s_1)_{\text{ideal}} \]

\[ = (0.1889)(0.05987 - (-0.002685)) + 2.1390 = 2.151 \text{ kJ/kg} \cdot \text{K} \]

Note that the temperature at the final state in this case was determined from

\[ T_2 = T_1 \frac{P_2}{P_1} Z_1 = (100 + 273 \text{ K}) \frac{8 \text{ MPa}}{1 \text{ MPa}} = 2888 \text{ K} \]

The solution using EES built-in property data is as follows:

\[ \nu_1 = 0.06885 \text{ m}^3/\text{kg} \]

\[ T_1 = 373 \text{ K} \]

\[ P_1 = 1 \text{ MPa} \]

\[ u_1 = -8.614 \text{ kJ/kg} \]

\[ P_2 = 8 \text{ MPa} \]

\[ \nu_2 = \nu_1 = 0.06885 \text{ m}^3/\text{kg} \]

\[ u_2 = 2754 \text{ kJ/kg} \]

\[ s_1 = -0.2464 \text{ kJ/kg.K} \]

\[ s_2 = 1.85 \text{ kJ/kg.K} \]

Then

\[ q_{\text{EES}} = u_2 - u_1 = 2754 - (-8.614) = 2763 \text{ kJ/kg} \]

\[ \Delta s_{\text{EES}} = (s_2 - s_1)_{\text{EES}} = s_2 - s_1 = 1.85 - (-0.2464) = 2.097 \text{ kJ/kg} \cdot \text{K} \]
12-71 For $\beta \geq 0$, it is to be shown that at every point of a single-phase region of an $h$s diagram, the slope of a constant-pressure line is greater than the slope of a constant-temperature line, but less than the slope of a constant-volume line.

**Analysis**  It is given that $\beta > 0$.

Using the Tds relation:  
\[
dh = T \, ds + \nu \, dP \quad \Rightarrow \quad \frac{dh}{ds} = T + \nu \frac{dP}{ds}
\]

1. $P$ = constant:  
\[
\left( \frac{\partial h}{\partial s} \right)_P = T
\]

2. $T$ = constant:  
\[
\left( \frac{\partial h}{\partial s} \right)_T = T + \nu \left( \frac{\partial P}{\partial s} \right)_T
\]

But the 4th Maxwell relation:  
\[
\left( \frac{\partial P}{\partial s} \right)_T = -\left( \frac{\partial T}{\partial \nu} \right)_P
\]

Substituting:  
\[
\left( \frac{\partial h}{\partial s} \right)_T = T - \nu \left( \frac{\partial T}{\partial \nu} \right)_P = T - \frac{1}{\beta}
\]

Therefore, the slope of $P$ = constant lines is greater than the slope of $T$ = constant lines.

3. $\nu$ = constant:  
\[
\left( \frac{\partial h}{\partial s} \right)_\nu = T + \nu \left( \frac{\partial P}{\partial s} \right)_\nu \quad (a)
\]

From the ds relation:  
\[
\frac{dP}{T} = \frac{c_v}{T} \, dT + \left( \frac{\partial P}{\partial T} \right)_\nu \, d\nu
\]

Divide by $dP$ holding $\nu$ constant:  
\[
\left( \frac{\partial S}{\partial P} \right)_\nu = \frac{c_v}{T} \left( \frac{\partial T}{\partial P} \right)_\nu \quad \text{or} \quad \left( \frac{\partial P}{\partial S} \right)_\nu = \frac{T}{c_v} \left( \frac{\partial P}{\partial T} \right)_\nu \quad (b)
\]

Using the properties $P$, $T$, $\nu$, the cyclic relation can be expressed as  
\[
\left( \frac{\partial P}{\partial T} \right)_\nu \left( \frac{\partial T}{\partial \nu} \right)_p \left( \frac{\partial \nu}{\partial P} \right)_T = -1 \quad \Rightarrow \quad \left( \frac{\partial P}{\partial T} \right)_\nu = -\left( \frac{\partial \nu}{\partial T} \right)_p \left( \frac{\partial P}{\partial \nu} \right)_T = (-\beta \nu) \left( \frac{1}{-\alpha \nu} \right) = \frac{\beta}{\alpha} \quad (c)
\]

where we used the definitions of $\alpha$ and $\beta$. Substituting (b) and (c) into (a),
\[
\left( \frac{\partial h}{\partial s} \right)_\nu = T + \nu \left( \frac{\partial P}{\partial s} \right)_\nu = T + \frac{T \beta \nu}{c_v \alpha} > T
\]

Here $\alpha$ is positive for all phases of all substances. $T$ is the absolute temperature that is also positive, so is $c_v$. Therefore, the second term on the right is always a positive quantity since $\beta$ is given to be positive. Then we conclude that the slope of $P$ = constant lines is less than the slope of $\nu$ = constant lines.
Using the cyclic relation and the first Maxwell relation, the other three Maxwell relations are to be obtained.

**Analysis** (1) Using the properties $P$, $s$, $v$, the cyclic relation can be expressed as

$$
\left( \frac{\partial P}{\partial s} \right) + \left( \frac{\partial s}{\partial v} \right)_P \left( \frac{\partial v}{\partial P} \right)_s = -1
$$

Substituting the first Maxwell relation,

$$
-\left( \frac{\partial T}{\partial v} \right)_s \left( \frac{\partial v}{\partial P} \right)_s = 1 \rightarrow \left( \frac{\partial T}{\partial P} \right)_s \left( \frac{\partial s}{\partial v} \right)_P = 1 \rightarrow \left( \frac{\partial T}{\partial P} \right)_s = \left( \frac{\partial v}{\partial s} \right)_P
$$

(2) Using the properties $T$, $v$, $s$, the cyclic relation can be expressed as

$$
\left( \frac{\partial T}{\partial v} \right)_P + \left( \frac{\partial v}{\partial s} \right)_T \left( \frac{\partial s}{\partial P} \right)_P = -1
$$

Substituting the first Maxwell relation,

$$
-\left( \frac{\partial P}{\partial s} \right)_v \left( \frac{\partial s}{\partial T} \right)_v = 1 \rightarrow \left( \frac{\partial P}{\partial T} \right)_v \left( \frac{\partial v}{\partial s} \right)_T = 1 \rightarrow \left( \frac{\partial v}{\partial s} \right)_T = \left( \frac{\partial P}{\partial T} \right)_v
$$

(3) Using the properties $P$, $T$, $v$, the cyclic relation can be expressed as

$$
\left( \frac{\partial P}{\partial T} \right)_v + \left( \frac{\partial T}{\partial v} \right)_P \left( \frac{\partial v}{\partial P} \right)_T = -1
$$

Substituting the third Maxwell relation,

$$
\left( \frac{\partial s}{\partial v} \right)_T \left( \frac{\partial T}{\partial P} \right)_v \left( \frac{\partial v}{\partial P} \right)_T = 1 \rightarrow \left( \frac{\partial v}{\partial P} \right)_T \left( \frac{\partial s}{\partial v} \right)_T = 1 \rightarrow \left( \frac{\partial v}{\partial P} \right)_T = \left( \frac{\partial T}{\partial v} \right)_P
$$

12-73 It is to be shown that the slope of a constant-pressure line on an $h$-$s$ diagram is constant in the saturation region and increases with temperature in the superheated region.

**Analysis** For $P =$ constant, $dP = 0$ and the given relation reduces to $dh = Tds$, which can also be expressed as

$$
\left( \frac{\partial h}{\partial s} \right)_P = T
$$

Thus the slope of the $P =$ constant lines on an $h$-$s$ diagram is equal to the temperature.

(a) In the saturation region, $T =$ constant for $P =$ constant lines, and the slope remains constant.

(b) In the superheat region, the slope increases with increasing temperature since the slope is equal temperature.
12-74 The relations for $\Delta u$, $\Delta h$, and $\Delta s$ of a gas that obeys the equation of state $(P+a/v^2)v = RT$ for an isothermal process are to be derived.

**Analysis** (a) For an isothermal process $dT = 0$ and the general relation for $\Delta u$ reduces to

$$\Delta u = u_2 - u_1 = \int_{v_1}^{v_2} c_v dT + \int_{v_1}^{v_2} \left( T \left( \frac{\partial P}{\partial T} \right)_v - P \right) dv = \int_{v_1}^{v_2} \left( T \left( \frac{\partial P}{\partial T} \right)_v - P \right) dv$$

For this gas the equation of state can be expressed as

$$P = \frac{RT}{v} - \frac{a}{v^2} \Rightarrow \left( \frac{\partial P}{\partial T} \right)_v = \frac{R}{v}$$

Thus,

$$T \left( \frac{\partial P}{\partial T} \right)_v - P = \frac{RT}{v} - \frac{RT}{v} + \frac{a}{v^2} = \frac{a}{v^2}$$

Substituting,

$$\Delta u = \int_{v_1}^{v_2} \frac{a}{v^2} dv = a \left( \frac{1}{v_1} - \frac{1}{v_2} \right)$$

(b) The enthalpy change $\Delta h$ is related to $\Delta u$ through the relation

$$\Delta h = \Delta u + P_2v_2 - P_1v_1$$

where

$$P_2v_2 - P_1v_1 = \left( RT - \frac{a}{v_2} \right) - \left( RT - \frac{a}{v_1} \right) = a \left( \frac{1}{v_1} - \frac{1}{v_2} \right)$$

Substituting,

$$\Delta h = 2a \left( \frac{1}{v_1} - \frac{1}{v_2} \right)$$

(c) For an isothermal process $dT = 0$ and the general relation for $\Delta s$ reduces to

$$\Delta s = s_2 - s_1 = \int_{v_1}^{v_2} \frac{c_v}{T} dT + \int_{v_1}^{v_2} \left( \frac{\partial P}{\partial T} \right)_v dv = \int_{v_1}^{v_2} \left( \frac{\partial P}{\partial T} \right)_v dv$$

Substituting $(\partial P/\partial T)_v = R/v$,

$$\Delta s = \int_{v_1}^{v_2} \frac{R}{v} dv = R \ln \frac{v_2}{v_1}$$
12-75 It is to be shown that
\[ c_v = -T \left( \frac{\partial v}{\partial T} \right)_s \left( \frac{\partial P}{\partial T} \right)_v \quad \text{and} \quad c_p = T \left( \frac{\partial P}{\partial T} \right)_s \left( \frac{\partial v}{\partial T} \right)_p \]

**Analysis** Using the definition of \( c_v \),
\[ c_v = -T \left( \frac{\partial v}{\partial T} \right)_s \left( \frac{\partial P}{\partial T} \right)_v \]
Substituting the first Maxwell relation
\[ \left( \frac{\partial s}{\partial T} \right)_v = \left( \frac{\partial v}{\partial T} \right)_s \]
\[ c_v = -T \left( \frac{\partial v}{\partial T} \right)_v \left( \frac{\partial P}{\partial T} \right)_v \]
Using the definition of \( c_p \),
\[ c_p = T \left( \frac{\partial P}{\partial T} \right)_s \left( \frac{\partial v}{\partial T} \right)_p \]
Substituting the second Maxwell relation
\[ \left( \frac{\partial s}{\partial T} \right)_p = \left( \frac{\partial P}{\partial T} \right)_s \]
\[ c_p = T \left( \frac{\partial P}{\partial T} \right)_s \left( \frac{\partial v}{\partial T} \right)_p \]

12-76 The \( C_p \) of nitrogen at 300 kPa and 400 K is to be estimated using the relation given and its definition, and the results are to be compared to the value listed in Table A-2b.

**Analysis (a)** We treat nitrogen as an ideal gas with \( R = 0.297 \text{ kJ/kg·K} \) and \( k = 1.397 \). Note that \( P T^{k/(k-1)} = C \) = constant for the isentropic processes of ideal gases. The \( c_p \) relation is given as
\[ c_p = T \left( \frac{\partial P}{\partial T} \right)_s \left( \frac{\partial v}{\partial T} \right)_p \]
\[ \text{Replaced by differences,} \]
\[ c_p = \left( \frac{\partial h}{\partial T} \right)_p \]
Substituting,
\[ c_p = T \left( \frac{kP}{T(k-1)} \right)_s \left( \frac{R}{P} \right)_p = \frac{kR}{k-1} = \frac{1.397(0.297 \text{ kJ/kg·K})}{1.397-1} = 1.045 \text{ kJ/kg·K} \]

(b) The \( c_p \) is defined as \( c_p = \left( \frac{\partial h}{\partial T} \right)_p \). Replacing the differentials by differences,
\[ c_p = \left( \frac{\Delta h}{\Delta T} \right)_{p=300\text{ kPa}} = \frac{h(410 \text{ K}) - h(390 \text{ K})}{410 - 390} = \frac{11,932 - 11,347}{10} = 0.585 \text{ kJ/kg·K} \]
(Compare: Table A-2b at 400 K → \( c_p = 1.044 \text{ kJ/kg·K} \))
12-77 The temperature change of steam and the average Joule-Thompson coefficient during a throttling process are to be estimated.

**Analysis** The enthalpy of steam at 4.5 MPa and \( T = 300 \, ^\circ\text{C} \) is \( h = 2944.2 \, \text{kJ/kg} \). Now consider a throttling process from this state to 2.5 MPa. The temperature of the steam at the end of this throttling process is

\[
\begin{align*}
P &= 2.5 \, \text{MPa} \\
h &= 2944.2 \, \text{kJ/kg} \\
T_2 &= 273.72 \, ^\circ\text{C}
\end{align*}
\]

Thus the temperature drop during this throttling process is

\[
\Delta T = T_2 - T_1 = 273.72 - 300 = -26.28 \, ^\circ\text{C}
\]

The average Joule-Thompson coefficient for this process is determined from

\[
\mu \left( \frac{\partial T}{\partial P} \right)_{h=3204.7 \, \text{kJ/kg}} = \frac{(273.72 - 300) \, ^\circ\text{C}}{(2.5 - 4.5) \, \text{MPa}} = 13.14 \, ^\circ\text{C}/\text{MPa}
\]

12-78 The initial state and the final temperature of argon contained in a rigid tank are given. The mass of the argon in the tank, the final pressure, and the heat transfer are to be determined using the generalized charts.

**Analysis** (a) The compressibility factor of argon at the initial state is determined from the generalized chart to be

\[
\begin{align*}
T_{R_i} &= \frac{T_1}{T_{cr}} = \frac{173}{151.0} = 1.146 \\
P_{R_i} &= \frac{P_1}{P_{cr}} = \frac{1}{4.86} = 0.206
\end{align*}
\]

Then,

\[
P \nu = ZRT \quad \Rightarrow \quad \nu = \frac{ZRT}{P} = \frac{(0.95)(0.2081 \, \text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(173 \, \text{K})}{1000 \, \text{kPa}} = 0.0342 \, \text{m}^3/\text{kg}
\]

\[
m = \frac{\nu}{\rho} = \frac{1.2 \, \text{m}^3}{0.0342 \, \text{m}^3/\text{kg}} = 35.1 \, \text{kg}
\]

(b) The specific volume of argon remains constant during this process, \( \nu_2 = \nu_1 \). Thus,

\[
\begin{align*}
T_{R_2} &= \frac{T_2}{T_{cr}} = \frac{273}{151.0} = 1.808 \\
\nu_{R_2} &= \frac{\nu_2}{RT_{cr} / P_{cr}} = \frac{0.0342 \, \text{m}^3/\text{kg}}{(0.2081 \, \text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(151 \, \text{K})(4860 \, \text{kPa})} = 5.29 \\
P_2 &= P_{R_2} P_{cr} = (0.315)(4860) = 1531 \, \text{kPa}
\end{align*}
\]

(c) The energy balance relation for this closed system can be expressed as

\[
E_{in} - E_{out} = \Delta E_{system}
\]

\[
Q_{in} = \Delta U = m(u_2 - u_1)
\]

\[
Q_{in} = m[h_2 - h_1 - (P_2 \nu_2 - P_1 \nu_1)] = m[h_2 - h_1 - R(Z_2 T_2 - Z_1 T_1)]
\]

where

\[
h_2 - h_1 = RT_{cr} [Z_h - Z_{b_2}] + (h_2 - h_1)_{\text{ideal}} = (0.2081)(151)(0.18 - 0) + 0.5203(0 - (-100)) = 57.69 \, \text{kJ/kg}
\]

Thus,

\[
Q_{in} = (35.1 \, \text{kg})(57.69 - (0.2081 \, \text{kJ/kg} \cdot \text{K})(0.99)(273) - (0.95)(173)) \, \text{kJ} = 1251 \, \text{kJ}
\]
12-79 Argon enters a turbine at a specified state and leaves at another specified state. Power output of the turbine and exergy destruction during this process are to be determined using the generalized charts.

Properties The gas constant and critical properties of Argon are \( R = 0.2081 \text{ kJ/kg.K} \), \( T_{cr} = 151 \text{ K} \), and \( P_{cr} = 4.86 \text{ MPa} \) (Table A-1).

Analysis (a) The enthalpy and entropy departures of argon at the specified states are determined from the generalized charts to be

\[
\begin{align*}
T_R &= \frac{T_1}{T_{cr}} = \frac{600}{151} = 3.97 \\
P_R &= \frac{P_1}{P_{cr}} = \frac{7}{4.86} = 1.44
\end{align*}
\]

Thus argon behaves as an ideal gas at turbine inlet. Also,

\[
\begin{align*}
T_1 &= 600 \text{ K} \\
V_1 &= 100 \text{ m/s} \\
\rho_1 &= 5 \text{ kg/s} \\
\end{align*}
\]

Thus, argon behaves as an ideal gas at turbine inlet. Also,

\[
\begin{align*}
T_R &= \frac{T_2}{T_{cr}} = \frac{280}{151} = 1.85 \\
P_R &= \frac{P_2}{P_{cr}} = \frac{1}{4.86} = 0.206
\end{align*}
\]

Thus,

\[
\begin{align*}
h_2 - h_1 &= RT_{cr} \left( Z_{h_2} - Z_{h_1} \right) + \left( h_2 - h_1 \right)_{ideal} \\
&= (0.2081)(151)(0 - 0.04) + 0.5203(280 - 600) = -167.8 \text{ kJ/kg}
\end{align*}
\]

The power output of the turbine is to be determined from the energy balance equation,

\[
\begin{align*}
\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system} &= 0 \text{ (steady)} \rightarrow \dot{E}_{in} = \dot{E}_{out} \\
\dot{m}(h_1 + \frac{V_1^2}{2}) = \dot{m}(h_2 + \frac{V_2^2}{2}) + \dot{Q}_{out} + \dot{W}_{out}
\end{align*}
\]

Substituting,

\[
\dot{W}_{out} = -5 \text{ kg/s} \left( -167.8 + \frac{(150 \text{ m/s})^2 - (100 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \right) - 60 \text{ kJ/s} = 747.8 \text{ kW}
\]

(b) Under steady conditions, the rate form of the entropy balance for the turbine simplifies to

\[
\begin{align*}
\dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen} &= \Delta \dot{S}_{system} = 0 \\
\dot{m}_{s_1} - \dot{m}_{s_2} - \frac{\dot{Q}_{out}}{T_{b_{out}}} + \dot{S}_{gen} &= 0 \rightarrow \dot{S}_{gen} = \dot{m}(s_2 - s_1) + \frac{\dot{Q}_{out}}{T_0}
\end{align*}
\]

The exergy destroyed during a process can be determined from an exergy balance or directly from its definition \( \dot{X}_{destroyed} = T_0 \dot{S}_{gen} \),

\[
\begin{align*}
\dot{X}_{destroyed} &= T_0 \dot{S}_{gen} = T_0 \left( \dot{m}(s_2 - s_1) + \frac{\dot{Q}_{out}}{T_0} \right)
\end{align*}
\]

where \( s_2 - s_1 = R(Z_{s_2} - Z_{s_1}) + (s_2 - s_1)_{ideal} \)

and \( (s_2 - s_1)_{ideal} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 0.5203 \ln \frac{280}{600} - 0.2081 \ln \frac{1}{7} = 0.0084 \text{ kJ/kg \cdot K} \)

Thus,

\[
\begin{align*}
s_2 - s_1 &= R(Z_{s_2} - Z_{s_1}) + (s_2 - s_1)_{ideal} = (0.2081)(0 - (0.02)) + 0.0084 = 0.0042 \text{ kJ/kg \cdot K}
\end{align*}
\]

Substituting,

\[
\begin{align*}
\dot{X}_{destroyed} &= (298 \text{ K}) \left( 5 \text{ kg/s} \right) \left( 0.0042 \text{ kJ/kg \cdot K} \right) + \frac{60 \text{ kW}}{298 \text{ K}} = 66.3 \text{ kW}
\end{align*}
\]
12-80 EES Problem 12-79 is reconsidered. The problem is to be solved assuming steam is the working fluid by using the generalized chart method and EES data for steam. The power output and the exergy destruction rate for these two calculation methods against the turbine exit pressure are to be plotted.

**Analysis** The problem is solved using EES, and the results are tabulated and plotted below.

"Input Data"

\[
\begin{align*}
T[1] &= 600 \text{ [K]} \\
P[1] &= 7000 \text{ [kPa]} \\
V[1] &= 100 \text{ [m/s]} \\
T[2] &= 455 \text{ [K]} \\
P[2] &= 1000 \text{ [kPa]} \\
V[2] &= 150 \text{ [m/s]} \\
Q_{\text{dot}}_{\text{out}} &= 60 \text{ [kW]} \\
T_o &= 25+273 \text{ [K]} \\
m_{\text{dot}} &= 5 \text{ [kg/s]} \\
\text{Name}$='\text{Steam}_iapws'$ \\
T_{\text{critical}} &= 647.3 \text{ [K]} \\
P_{\text{critical}} &= 22090 \text{ [kPa]} \\
\text{Fluid}$='\text{H2O}' \\
R_u &= 8.314 \\
M &= \text{molarmass(Fluid$)} \\
R &= \text{R_u/M}
\end{align*}
\]

"****** IDEAL GAS SOLUTION ******"

"State 1"

\[
\begin{align*}
h_{\text{ideal}}[1] &= \text{enthalpy(Fluid$, T=T[1])} \quad \text{"Enthalpy of ideal gas"} \\
s_{\text{ideal}}[1] &= \text{entropy(Fluid$, T=T[1], P=P[1])} \quad \text{"Entropy of ideal gas"} \\
\end{align*}
\]

"State 2"

\[
\begin{align*}
h_{\text{ideal}}[2] &= \text{enthalpy(Fluid$, T=T[2])} \quad \text{"Enthalpy of ideal gas"} \\
s_{\text{ideal}}[2] &= \text{entropy(Fluid$, T=T[2], P=P[2])} \quad \text{"Entropy of ideal gas"} \\
\end{align*}
\]

"Conservation of Energy, Steady-flow:"

\[
\begin{align*}
E_{\text{dot in}} &= E_{\text{dot out}} \\
m_{\text{dot}} \cdot (h_{\text{ideal}}[1]+V[1]^2/2) \cdot \text{convert(m^2/s^2,kJ/kg)} &= m_{\text{dot}} \cdot (h_{\text{ideal}}[2]+V[2]^2/2) \cdot \text{convert(m^2/s^2,kJ/kg)} + Q_{\text{dot}}_{\text{out}} + W_{\text{dot}}_{\text{out ideal}}
\end{align*}
\]

"Second Law analysis:"

\[
\begin{align*}
S_{\text{dot in}} + S_{\text{dot out}} + S_{\text{dot gen}} &= 0 \\
m_{\text{dot}} \cdot s_{\text{ideal}}[1] - m_{\text{dot}} \cdot s_{\text{ideal}}[2] - Q_{\text{dot}}_{\text{out}}/T_o + S_{\text{dot gen ideal}} &= 0
\end{align*}
\]

"Exergy Destroyed:"

\[
X_{\text{dot destroyed ideal}} = T_o \cdot S_{\text{dot gen ideal}}
\]

"****** COMPRESSABILITY CHART SOLUTION ******"

"State 1"

\[
\begin{align*}
Tr[1] &= T[1]/T_{\text{critical}} \\
P[1] &= P[1]/P_{\text{critical}} \\
Z[1] &= \text{COMPRESS}(Tr[1], Pr[1]) \\
\text{DELTAh}[1] &= \text{ENTHDEP}(Tr[1], Pr[1]) \cdot R \cdot T_{\text{critical}} \quad \text{"Enthalpy departure"} \\
h_{\text{chart}}[1] &= h_{\text{ideal}}[1] - \text{DELTAh}[1] \quad \text{"Enthalpy of real gas using charts"} \\
\text{DELTAs}[1] &= \text{ENTRDEP}(Tr[1], Pr[1]) \cdot R \quad \text{"Entropy departure"} \\
s_{\text{chart}}[1] &= s_{\text{ideal}}[1] - \text{DELTAs}[1] \quad \text{"Entropy of real gas using charts"}
\end{align*}
\]

"State 2"
\( T_{r2} = \frac{T[2]}{T_{critical}} \)
\( P_{r2} = \frac{P[2]}{P_{critical}} \)
\( Z[2] = \text{COMPRESS}(T[2], P[2]) \)
\( \Delta T[2] = \text{ENTHDEP}(T[2], P[2]) \cdot R \cdot T_{critical} \) "Enthalpy departure"
\( \Delta T[2] = \text{ENTRDEP}(T[2], P[2]) \cdot R \) "Entropy departure"
\( h_{chart}[2] = h_{ideal}[2] - \Delta h[2] \) "Enthalpy of real gas using charts"
\( s_{chart}[2] = s_{ideal}[2] - \Delta s[2] \) "Entropy of real gas using charts"

"Conservation of Energy, Steady-flow:
"\( E_{dot\_in} = E_{dot\_out} \)"

\[ m_{dot} \cdot (h_{chart}[1] + \frac{\text{Vel}[1]^2}{2} \cdot \text{convert(m}^2/\text{s}^2, \text{kJ/kg})) = m_{dot} \cdot (h_{chart}[2] + \frac{\text{Vel}[2]^2}{2} \cdot \text{convert(m}^2/\text{s}^2, \text{kJ/kg})) + Q_{dot\_out} + W_{dot\_out\_chart} \]

"Second Law analysis:
"\( S_{dot\_in} - S_{dot\_out} + S_{dot\_gen} = 0 \)"
\[ m_{dot} \cdot s_{chart}[1] - m_{dot} \cdot s_{chart}[2] - \frac{Q_{dot\_out}}{T_o} + S_{dot\_gen\_chart} = 0 \]

"Exergy Destroyed:
\[ X_{dot\_destroyed\_chart} = T_o \cdot S_{dot\_gen\_chart}[\text{kJW}] \]

"***** SOLUTION USING EES BUILT-IN PROPERTY DATA *****"

"At state 1"
\[ h_{ees}[1] = \text{enthalpy(Name$, T=T[1], P=P[1])} \]
\[ s_{ees}[1] = \text{entropy(Name$, T=T[1], P=P[1])} \]

"At state 2"
\[ h_{ees}[2] = \text{enthalpy(Name$, T=T[2], P=P[2])} \]
\[ s_{ees}[2] = \text{entropy(Name$, T=T[2], P=P[2])} \]

"Conservation of Energy, Steady-flow:
"\( E_{dot\_in} = E_{dot\_out} \)"

\[ m_{dot} \cdot (h_{ees}[1] + \frac{\text{Vel}[1]^2}{2} \cdot \text{convert(m}^2/\text{s}^2, \text{kJ/kg})) = m_{dot} \cdot (h_{ees}[2] + \frac{\text{Vel}[2]^2}{2} \cdot \text{convert(m}^2/\text{s}^2, \text{kJ/kg})) + Q_{dot\_out} + W_{dot\_out\_ees} \]

"Second Law analysis:
"\( S_{dot\_in} - S_{dot\_out} + S_{dot\_gen} = 0 \)"
\[ m_{dot} \cdot s_{ees}[1] - m_{dot} \cdot s_{ees}[2] - \frac{Q_{dot\_out}}{T_o} + S_{dot\_gen\_ees} = 0 \]

"Exergy Destroyed:
\[ X_{dot\_destroyed\_ees} = T_o \cdot S_{dot\_gen\_ees} \]
<table>
<thead>
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<th>$P_2$ [kPa]</th>
<th>$T_2$ [K]</th>
<th>$W_{out,chart}$ [kW]</th>
<th>$W_{out,ees}$ [kW]</th>
<th>$W_{out,ideal}$ [kW]</th>
<th>$X_{destroyed,chart}$ [kW]</th>
<th>$X_{destroyed,ees}$ [kW]</th>
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</table>
Argon gas enters a turbine at a specified state and leaves at another specified state. The power output of the turbine and the exergy destruction associated with the process are to be determined using the generalized charts.

**Properties**  
The gas constant and critical properties of argon are:

- \( R = 0.04971 \text{ Btu/lbm.R} \)
- \( T_{cr} = 272 \text{ R} \)
- \( P_{cr} = 705 \text{ psia} \) (Table A-1E).

**Analysis**  
(a) The enthalpy and entropy departures of argon at the specified states are determined from the generalized charts to be:

\[
\begin{align*}
T_R &= T_1 = \frac{1000}{272} \approx 3.68 \\
P_R &= \frac{P_1}{705} = \frac{1000}{705} = 1.418
\end{align*}
\]

Thus, argon behaves as an ideal gas at turbine inlet. Also,

\[
\begin{align*}
T_{R_2} &= T_2 = \frac{500}{272} = 1.838 \\
P_{R_2} &= \frac{P_2}{705} = \frac{150}{705} = 0.213
\end{align*}
\]

Thus,

\[
h_2 - h_1 = RT_{cr} (Z_{h_2} - Z_{h_1}) + (h_2 - h_1)_{ideal} = (0.04971)(272)(0 - 0.04) + 0.1253(500 - 1000) = -63.2 \text{ Btu/lbm}
\]

The power output of the turbine is to be determined from the energy balance equation:

\[
\dot{E}_{in} - \dot{E}_{out} = \Delta E_{system} = 0 \quad \text{(steady)} \quad \Rightarrow \dot{E}_{in} = \dot{E}_{out}
\]

\[
\dot{m}(h_1 + V_1^2/2) = \dot{m}(h_2 + V_2^2/2) + \dot{Q}_{out} + W_{out} \quad \Rightarrow \dot{W}_{out} = -\dot{m} (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} - \dot{Q}_{out}
\]

\[
W_{out} = -(12 \text{ lbm/s}) \left( -63.2 + \frac{(450 \text{ ft/s})^2 - (300 \text{ ft/s})^2}{2} \left( \frac{1 \text{ Btu/lbm}}{25,037 \text{ ft}^2/\text{s}^2} \right) \right) - 80 \text{ Btu/s}
\]

\[
= 651.4 \text{ Btu/s} = 922 \text{ hp}
\]

(b) Under steady conditions, the rate form of the entropy balance for the turbine simplifies to:

\[
\dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen} = \Delta S_{system} \Rightarrow \dot{S}_{gen} = 0
\]

\[
\dot{m}s_1 - \dot{m}s_2 = \frac{\dot{Q}_{out}}{T_{p, out}} + \dot{S}_{gen} = 0 \quad \Rightarrow \dot{S}_{gen} = \dot{m}(s_2 - s_1) + \frac{\dot{Q}_{out}}{T_0}
\]

The exergy destroyed during a process can be determined from an exergy balance or directly from its definition:

\[
\dot{X}_{destroyed} = T_0 \dot{S}_{gen}
\]

where

\[
s_2 - s_1 = R(Z_{s_2} - Z_{s_1}) + (s_2 - s_1)_{ideal}
\]

and

\[
(s_2 - s_1)_{ideal} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 0.1253 \ln \frac{500}{1000} - 0.04971 \ln \frac{150}{1000} = 0.00745 \text{ Btu/lbm} \cdot \text{R}
\]

Thus, \( s_2 - s_1 = R(Z_{s_1} - Z_{s_1}) + (s_2 - s_1)_{ideal} = (0.04971)(0 - 0.02) + 0.00745 = 0.00646 \text{ Btu/lbm} \cdot \text{R} \)

Substituting,

\[
\dot{X}_{destroyed} = \left(535 R \left(12 \text{ lbm/s} \left(80 \text{ Btu/s} / \text{lbm} \cdot \text{R} \right) + \frac{80 \text{ Btu/s}}{535 \text{ R}} \right) \right) = 121.5 \text{ Btu/s}
\]
An adiabatic storage tank that is initially evacuated is connected to a supply line that carries nitrogen. A valve is opened, and nitrogen flows into the tank. The final temperature in the tank is to be determined by treating nitrogen as an ideal gas and using the generalized charts, and the results are to be compared to the given actual value.

**Assumptions**

1. Uniform flow conditions exist.
2. Kinetic and potential energies are negligible.

**Analysis**

We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy $h$ and internal energy $u$, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \rightarrow m_i = m_2$ (since $m_{\text{out}} = m_{\text{initial}} = 0$)

Energy balance: $E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}} \rightarrow 0 + m_i h_i = m_2 u_2$

Combining the two balances: $u_2 = h_i$

(a) From the ideal gas property table of nitrogen, at 225 K we read

$\overline{u}_i = \overline{h}_i = \overline{h}_{\text{at} 225\ K} = 6,537 \text{ kJ/kmol}$

The temperature that corresponds to this $\overline{u}_i$ value is

$T_2 = 314.8 \text{ K} \quad (7.4\% \text{ error})$

(b) Using the generalized enthalpy departure chart, $h_i$ is determined to be

$$T_{R,i} = \frac{T_i}{T_{cr}} = \frac{225}{126.2} = 1.78 \quad Z_{h,i} = \frac{\overline{h}_{i,\text{ideal}} - \overline{h}_i}{R_u T_{cr}} = 0.9 \quad \text{(Fig. A-29)}$$

Thus,

$$\overline{h}_i = \overline{h}_{i,\text{ideal}} - 0.9 R_u T_{cr} = 6,537 - (0.9)(8.314)(126.2) = 5,593 \text{ kJ/kmol}$$

and

$$\overline{u}_2 = \overline{h}_i = 5,593 \text{ kJ/kmol}$$

Try $T_2 = 280 \text{ K}$. Then at $P_{R2} = 2.95$ and $T_{R2} = 2.22 \text{ we read } Z_2 = 0.98$ and $(\overline{h}_{2,\text{ideal}} - \overline{h}_2) / R_u T_{cr} = 0.55$

Thus,

$$\overline{h}_2 = \overline{h}_{2,\text{ideal}} - 0.55 R_u T_{cr} = 8,141 - (0.55)(8.314)(126.2) = 7,564 \text{ kJ/kmol}$$

$$\overline{u}_2 = \overline{h}_2 - ZR_u T_2 = 7,564 - (0.98)(8.314)(280) = 5,283 \text{ kJ/kmol}$$

Try $T_2 = 300 \text{ K}$. Then at $P_{R2} = 2.95$ and $T_{R2} = 2.38 \text{ we read } Z_2 = 1.0$ and $(\overline{h}_{2,\text{ideal}} - \overline{h}_2) / R_u T_{cr} = 0.50$

Thus,

$$\overline{h}_2 = \overline{h}_{2,\text{ideal}} - 0.50 R_u T_{cr} = 8,723 - (0.50)(8.314)(126.2) = 8,198 \text{ kJ/kmol}$$

$$\overline{u}_2 = \overline{h}_2 - ZR_u T_2 = 8,198 - (1.0)(8.314)(300) = 5,704 \text{ kJ/kmol}$$

By linear interpolation,

$T_2 = 294.7 \text{ K} \quad (0.6\% \text{ error})$
It is to be shown that \( \frac{d\nu}{\nu} = \beta dT - \alpha dP \). Also, a relation is to be obtained for the ratio of specific volumes \( \nu_2/\nu_1 \) as a homogeneous system undergoes a process from state 1 to state 2.

**Analysis**

We take \( \nu = \nu(P, T) \). Its total differential is

\[
d\nu = \left( \frac{\partial \nu}{\partial T} \right)_P dT + \left( \frac{\partial \nu}{\partial P} \right)_T dP
\]

Dividing by \( \nu \),

\[
\frac{d\nu}{\nu} = \frac{1}{\nu} \left( \frac{\partial \nu}{\partial T} \right)_P dT + \frac{1}{\nu} \left( \frac{\partial \nu}{\partial P} \right)_T dP
\]

Using the definitions of \( \alpha \) and \( \beta \),

\[
\frac{d\nu}{\nu} = \beta dT - \alpha dP
\]

Taking \( \alpha \) and \( \beta \) to be constants, integration from 1 to 2 yields

\[
\ln \frac{\nu_2}{\nu_1} = \beta(T_2 - T_1) - \alpha(P_2 - P_1)
\]

which is the desired relation.

It is to be shown that \( \frac{d\nu}{\nu} = \beta dT - \alpha dP \). Also, a relation is to be obtained for the ratio of specific volumes \( \nu_2/\nu_1 \) as a homogeneous system undergoes an isobaric process from state 1 to state 2.

**Analysis**

We take \( \nu = \nu(P, T) \). Its total differential is

\[
d\nu = \left( \frac{\partial \nu}{\partial T} \right)_P dT + \left( \frac{\partial \nu}{\partial P} \right)_T dP
\]

which, for a constant pressure process, reduces to

\[
d\nu = \left( \frac{\partial \nu}{\partial T} \right)_P dT
\]

Dividing by \( \nu \),

\[
\frac{d\nu}{\nu} = \frac{1}{\nu} \left( \frac{\partial \nu}{\partial T} \right)_P dT
\]

Using the definition of \( \beta \),

\[
\frac{d\nu}{\nu} = \beta dT
\]

Taking \( \beta \) to be a constant, integration from 1 to 2 yields

\[
\ln \frac{\nu_2}{\nu_1} = \beta(T_2 - T_1) = \ln \left[ \exp \left( \beta(T_2 - T_1) \right) \right]
\]

or

\[
\frac{\nu_2}{\nu_1} = \exp \left[ \beta(T_2 - T_1) \right]
\]

which is the desired relation.
The volume expansivity of water is given. The change in volume of water when it is heated at constant pressure is to be determined.

**Properties**

The volume expansivity of water is given to be $0.207 \times 10^{-6} \text{ K}^{-1}$ at $20^\circ \text{C}$.

**Analysis**

We take $\nu = \nu(P, T)$. Its total differential is

$$d\nu = \left(\frac{\partial \nu}{\partial T}\right)_P dT + \left(\frac{\partial \nu}{\partial P}\right)_T dP$$

which, for a constant pressure process, reduces to

$$d\nu = \left(\frac{\partial \nu}{\partial T}\right)_P dT$$

Dividing by $\nu$ and using the definition of $\beta$,

$$\frac{d\nu}{\nu} = \frac{1}{\nu} \left(\frac{\partial \nu}{\partial T}\right)_P dT = \beta dT$$

Taking $\beta$ to be a constant, integration from 1 to 2 yields

$$\ln \frac{\nu_2}{\nu_1} = \beta (T_2 - T_1)$$

or

$$\frac{\nu_2}{\nu_1} = \exp[\beta (T_2 - T_1)]$$

Substituting the given values and noting that for a fixed mass $\frac{\nu_2}{\nu_1} = \nu_2/\nu_1$,

$$\nu_2 = \nu_1 \exp[\beta (T_2 - T_1)] = \left(1 \text{ m}^3\right) \exp\left[\left(0.207 \times 10^{-6} \text{ K}^{-1}\right)(30 - 10)\text{C}\right]$$

$$= 1.00000414 \text{ m}^3$$

Therefore,

$$\Delta \nu = \nu_2 - \nu_1 = 1.00000414 - 1 = 0.00000414 \text{ m}^3 = 4.14 \text{ cm}^3$$
The volume expansivity of copper is given at two temperatures. The percent change in the volume of copper when it is heated at atmospheric pressure is to be determined.

**Properties** The volume expansivity of copper is given to be \(49.2 \times 10^{-6} \text{ K}^{-1}\) at 300 K, and be \(54.2 \times 10^{-6} \text{ K}^{-1}\) at 500 K.

**Analysis** We take \(v = v(P, T)\). Its total differential is

\[ dv = \left( \frac{\partial v}{\partial T} \right)_P dT + \left( \frac{\partial v}{\partial P} \right)_T dP \]

which, for a constant pressure process, reduces to

\[ dv = \left( \frac{\partial v}{\partial T} \right)_P dT \]

Dividing by \(v\) and using the definition of \(\beta\),

\[ \frac{d\nu}{\nu} = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_P dT = \beta dT \]

Taking \(\beta\) to be a constant, integration from 1 to 2 yields

\[ \ln \frac{\nu_2}{\nu_1} = \beta(T_2 - T_1) \]

or

\[ \frac{\nu_2}{\nu_1} = \exp[\beta(T_2 - T_1)] \]

The average value of \(\beta\) is

\[ \beta_{\text{ave}} = \left( \beta_1 + \beta_2 \right)/2 = \left(49.2 \times 10^{-6} + 54.2 \times 10^{-6} \right)/2 = 51.7 \times 10^{-6} \text{ K}^{-1} \]

Substituting the given values,

\[ \frac{\nu_2}{\nu_1} = \exp[\beta(T_2 - T_1)] = \exp\left[(51.7 \times 10^{-6} \text{ K}^{-1})(500 - 300)\text{K}\right] = 1.0104 \]

Therefore, the volume of copper block will increase by **1.04 percent**.
12-87 It is to be shown that the position of the Joule-Thompson coefficient inversion curve on the $T$-$P$ plane is given by $(\partial Z/\partial T)_p = 0$.

**Analysis** The inversion curve is the locus of the points at which the Joule-Thompson coefficient $\mu$ is zero,

$$\mu = \frac{1}{c_p} \left( T \left( \frac{\partial \nu}{\partial T} \right)_p - \nu \right) = 0$$

which can also be written as

$$T \left( \frac{\partial \nu}{\partial T} \right)_p - \frac{ZRT}{P} = 0 \quad (a)$$

since it is given that

$$\nu = \frac{ZRT}{P} \quad (b)$$

Taking the derivative of $(b)$ with respect to $T$ holding $P$ constant gives

$$\left( \frac{\partial \nu}{\partial T} \right)_p = \left( \frac{\partial (ZRT/P)}{\partial T} \right)_p = \frac{R}{P} \left( \frac{\partial Z}{\partial T} \right)_p + Z$$

Substituting in $(a)$,

$$\frac{TR}{P} \left( \frac{\partial Z}{\partial T} \right)_p + Z - \frac{ZRT}{P} = 0$$

$$T \left( \frac{\partial Z}{\partial T} \right)_p + Z - Z = 0$$

$$\left( \frac{\partial Z}{\partial T} \right)_p = 0$$

which is the desired relation.
It is to be shown that for an isentropic expansion or compression process $P\nu^k = \text{constant}$. It is also to be shown that the isentropic expansion exponent $k$ reduces to the specific heat ratio $c_p/c_v$ for an ideal gas.

**Analysis** We note that $ds = 0$ for an isentropic process. Taking $s = s(P, \nu)$, the total differential $ds$ can be expressed as

$$ds = \left(\frac{\partial s}{\partial P}\right)_\nu dP + \left(\frac{\partial s}{\partial \nu}\right)_P d\nu = 0 \quad (a)$$

We now substitute the Maxwell relations below into $(a)$

$$\left(\frac{\partial s}{\partial P}\right)_\nu = -\left(\frac{\partial \nu}{\partial T}\right)_s$$

and

$$\left(\frac{\partial s}{\partial \nu}\right)_P = \left(\frac{\partial P}{\partial T}\right)_s$$

to get

$$-\left(\frac{\partial \nu}{\partial T}\right)_s dP + \left(\frac{\partial P}{\partial P}\right)_s d\nu = 0$$

Rearranging,

$$dP - \left(\frac{\partial T}{\partial \nu}\right)_s \left(\frac{\partial P}{\partial \nu}\right)_s d\nu = 0 \quad \rightarrow \quad dP - \left(\frac{\partial P}{\partial \nu}\right)_s d\nu = 0$$

Dividing by $P$,

$$\frac{dP}{P} - \frac{1}{P} \left(\frac{\partial P}{\partial \nu}\right)_s d\nu = 0 \quad (b)$$

We now define isentropic expansion exponent $k$ as

$$k = -\frac{\nu}{P} \left(\frac{\partial P}{\partial \nu}\right)_s$$

Substituting in $(b)$,

$$\frac{dP}{P} + k \frac{d\nu}{\nu} = 0$$

Taking $k$ to be a constant and integrating,

$$\ln P + k \ln \nu = \text{constant} \quad \rightarrow \quad \ln P \nu^k = \text{constant}$$

Thus,

$$P\nu^k = \text{constant}$$

To show that $k = c_p/c_v$ for an ideal gas, we write the cyclic relations for the following two groups of variables:

$$(s, T, \nu) \rightarrow \left(\frac{\partial s}{\partial T}\right)_\nu \left(\frac{\partial \nu}{\partial s}\right)_T \left(\frac{\partial T}{\partial \nu}\right)_s = -1 \quad \rightarrow \quad c_v \left(\frac{\partial \nu}{\partial s}\right)_T \left(\frac{\partial T}{\partial \nu}\right)_s = -1 \quad (c)$$

$$(s, T, P) \rightarrow \left(\frac{\partial s}{\partial T}\right)_P \left(\frac{\partial P}{\partial s}\right)_T \left(\frac{\partial T}{\partial P}\right)_s = -1 \quad \rightarrow \quad c_p \left(\frac{\partial P}{\partial s}\right)_T \left(\frac{\partial T}{\partial P}\right)_s = -1 \quad (d)$$

where we used the relations

$$c_v = T \left(\frac{\partial s}{\partial T}\right)_\nu$$

and

$$c_p = T \left(\frac{\partial s}{\partial T}\right)_P$$

Setting Eqs. $(c)$ and $(d)$ equal to each other,

$$\frac{c_p}{c_v} \left(\frac{\partial P}{\partial s}\right)_T \left(\frac{\partial s}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_s = \frac{c_v}{c_p} \left(\frac{\partial s}{\partial s}\right)_T \left(\frac{\partial T}{\partial s}\right)_P = \left(\frac{\partial \nu}{\partial s}\right)_P \left(\frac{\partial s}{\partial s}\right)_T \left(\frac{\partial T}{\partial \nu}\right)_s$$

or,

$$\frac{c_p}{c_v} = \frac{\left(\frac{\partial s}{\partial T}\right)_T \left(\frac{\partial s}{\partial s}\right)_P \left(\frac{\partial T}{\partial P}\right)_s}{\left(\frac{\partial P}{\partial \nu}\right)_s \left(\frac{\partial s}{\partial \nu}\right)_P \left(\partial T\right)}$$

but

$$\left(\frac{\partial \nu}{\partial P}\right)_T = \frac{\partial (RT/P)}{\partial P} = -\frac{\nu}{P}$$

Substituting,

$$\frac{c_p}{c_v} = -\frac{\nu}{P} \left(\frac{\partial P}{\partial \nu}\right)_s = k$$

which is the desired relation.
**12-89 EES** The work done by the refrigerant 134a as it undergoes an isothermal process in a closed system is to be determined using the tabular (EES) data and the generalized charts.

**Analysis** The solution using EES built-in property data is as follows:

\[
\begin{align*}
T_1 &= 60^\circ C \quad u_1 = 135.65 \text{ kJ/kg} \\
P_1 &= 3 \text{ MPa} \quad s_1 = 0.4828 \text{ kJ/kg.K} \\
T_2 &= 60^\circ C \quad u_2 = 280.35 \text{ kJ/kg} \\
P_2 &= 0.1 \text{ MPa} \quad s_2 = 1.2035 \text{ kJ/kg.K} \\
\Delta s_{\text{EES}} &= s_2 - s_1 = 1.2035 - 0.4828 = 0.7207 \text{ kJ/kg.K} \\
qu_{\text{EES}} &= T_1 \Delta s_{\text{EES}} = (60 + 273.15 \text{ K})(0.7207 \text{ kJ/kg.K}) = 240.11 \text{ kJ/kg} \\
w_{\text{EES}} &= q_{\text{EES}} - (u_2 - u_1) = 240.1 - (280.35 - 135.65) = 95.40 \text{ kJ/kg} \\
\end{align*}
\]

For the generalized chart solution we first determine the following factors using EES as

\[
\begin{align*}
T_{R1} &= \frac{T_1}{T_{cr}} = \frac{333.15}{374.2} = 0.8903 \\
P_{R1} &= \frac{P_1}{P_{cr}} = \frac{3}{4.059} = 0.7391 \\
T_{R2} &= \frac{T_2}{T_{cr}} = \frac{333.15}{374.2} = 0.8903 \\
P_{R2} &= \frac{P_2}{P_{cr}} = \frac{0.1}{4.059} = 0.02464 \\
\end{align*}
\]

Then,

\[
\begin{align*}
\Delta h_1 &= Z_{h1}RT_{cr} = (4.475)(0.08148 \text{ kJ/kg.K})(374.2 \text{ K}) = 136.43 \text{ kJ/kg} \\
\Delta s_1 &= Z_{s1}R = (4.383)(0.08148 \text{ kJ/kg.K}) = 0.3572 \text{ kJ/kg.K} \\
\Delta h_2 &= Z_{h2}RT_{cr} = (0.03091)(0.08148 \text{ kJ/kg.K})(374.2 \text{ K}) = 0.94 \text{ kJ/kg} \\
\Delta s_2 &= Z_{s2}R = (0.02281)(0.08148 \text{ kJ/kg.K}) = 0.001858 \text{ kJ/kg.K} \\
\Delta s_{\text{ideal}} &= R \ln \frac{P_2}{P_1} = (0.08148 \text{ kJ/kg.K}) \ln \left( \frac{0.1}{3} \right) = 0.2771 \text{ kJ/kg.K} \\
\Delta s_{\text{chart}} &= \Delta s_{\text{ideal}} - (\Delta s_2 - \Delta s_1) = 0.2771 - (0.001858 - 0.3572) = 0.6324 \text{ kJ/kg.K} \\
qu_{\text{chart}} &= T_1 \Delta s_{\text{chart}} = (60 + 273.15 \text{ K})(0.6324 \text{ kJ/kg.K}) = 210.70 \text{ kJ/kg} \\
\Delta u_{\text{chart}} &= \Delta h_{\text{ideal}} - (\Delta h_2 - \Delta h_1) - (Z_2RT_2 - Z_1RT_1) \\
&= 0 - (0.94 - 136.43) - \left[ (0.988)(0.08148)(333) - (0.1292)(0.08148)(333) \right] = 112.17 \text{ kJ/kg} \\
w_{\text{chart}} &= q_{\text{chart}} - \Delta u_{\text{chart}} = 210.70 - 112.17 = 98.53 \text{ kJ/kg} \\
\end{align*}
\]

The copy of the EES solution of this problem is given next.
"Input data"
T_critical=T_CRIT(R134a) "[K]"
P_critical=P_CRIT(R134a) "[kpa]"
T[1]=60+273.15"[K]"
T[2]=T[1]"[K]"
P[1]=3000"[kPa]"
P[2]=100"[kPa]"
R_u=8.314"[kJ/kmol-K]"
M=molarmass(R134a)
R=R_u/M"[kJ/kg-K]"

***** SOLUTION USING EES BUILT-IN PROPERTY DATA *****

"For the isothermal process, the heat transfer is T*(s[2] - s[1]):"
DELTAs_EES=(entropy(R134a,T=T[2],P=P[2])-entropy(R134a,T=T[1],P=P[1]))
q_EES=T[1]*DELTAs_EES
s_2=entropy(R134a,T=T[2],P=P[2])
s_1=entropy(R134a,T=T[1],P=P[1])

"Conservation of energy for the closed system:"
DELTAu_EES=intEnergy(R134a,T=T[2],p=P[2])-intEnergy(R134a,T=T[1],P=P[1])
q_EES-w_EES=DELTAu_EES
u_1=intEnergy(R134a,T=T[1],P=P[1])
u_2=intEnergy(R134a,T=T[2],p=P[2])

***** COMPRESSABILITY CHART SOLUTION *****

"State 1"
Tr[1]=T[1]/T_critical
pr[1]=p[1]/p_critical
Z[1]=COMPRESS(Tr[1], Pr[1])
DELTAh[1]=ENTHDEP(Tr[1], Pr[1])*R*T_critical"Enthalpy departure"
Z_h1=ENTHDEP(Tr[1], Pr[1])
DELTAas[1]=ENTRDEP(Tr[1], Pr[1])*R "Entropy departure"
Z_s1=ENTRDEP(Tr[1], Pr[1])

"State 2"
Z[2]=COMPRESS(Tr[2], Pr[2])
DELTAh[2]=ENTHDEP(Tr[2], Pr[2])*R*T_critical"Enthalpy departure"
Z_h2=ENTHDEP(Tr[2], Pr[2])
DELTAas[2]=ENTRDEP(Tr[2], Pr[2])*R "Entropy departure"
Z_s2=ENTRDEP(Tr[2], Pr[2])

"Entropy Change"
DELTAas_ideal= -R*ln(P[2]/P[1])
DELTAas_chart=DELTAas_ideal-(DELTAas[2]-DELTAas[1])

"For the isothermal process, the heat transfer is T*(s[2] - s[1]):"
q_chart=T[1]*DELTAas_chart

"Conservation of energy for the closed system:"
DELTAh_ideal=0
DELTAu_chart=DELTAh_ideal-(DELTAh[2]-DELTAh[1])-(Z[2]*R*T[2]-Z[1]*R*T[1])
q_chart-w_chart=DELTAu_chart
SOLUTION

DELTAh[1]=136.43
DELTAh[2]=0.94
DELTAh_ideal=0
DELTAu[1]=0.3572
DELTAu[2]=0.001858
DELTAu_chart=0.6324 [kJ/kg-K]
DELTAu_EES=0.7207 [kJ/kg-K]
DELTAu_ideal=0.2771 [kJ/kg-K]
DELTAu_chart=112.17
DELTAu_EES=144.7

M=102 [kg/kmol]
P[1]=3000 [kPa]
P[2]=100 [kPa]
Pr[1]=0.7391
Pr[2]=0.02464
P_critical=4059 [kPa]
q_chart=210.70 [kJ/kg]
q_EES=240.11 [kJ/kg]
R=0.08148 [kJ/kg-K]

R_u=8.314 [kJ/kmol-K]
s_1=0.4828 [kJ/kg-K]
s_2=1.2035 [kJ/kg-K]
T[1]=333.2 [K]
T[2]=333.2 [K]
Tr[1]=0.8903
Tr[2]=0.8903
T_critical=374.2 [K]
u_1=135.65 [kJ/kg]
u_2=280.35 [kJ/kg]
w_chart=98.53 [kJ/kg]
w_EES=95.42 [kJ/kg]
Z[1]=0.1292
Z[2]=0.988
Z_h1=4.475
Z_h2=0.03091
Z_s1=4.383
Z_s2=0.02281
The heat transfer, work, and entropy changes of methane during a process in a piston-cylinder device are to be determined assuming ideal gas behavior, using generalized charts, and real fluid (EES) data.

**Analysis** The ideal gas solution: (Properties are obtained from EES)

State 1: $h_1 = 10^0C \rightarrow h_1 = -4492 \text{kJ/kg}$

$T_1 = 10^0C, P_1 = 4 \text{MPa} \rightarrow s_1 = 10.22 \text{kJ/kg.K}$

$u_1 = h_1 - RT_1 = (-4492) - (0.5182)(100 + 273.15) = -4685 \text{kJ/kg}$

$\nu_1 = R \frac{T_1}{P_1} = (0.5182 \text{kJ/kg.K})(\frac{100 + 273.15 \text{K}}{4000 \text{kPa}}) = 0.04834 \text{m}^3/\text{kg}$

$T_2 = 350^0C \rightarrow h_2 = -3770 \text{kJ/kg}$

$T_2 = 350^0C, P_2 = 4 \text{MPa} \rightarrow s_2 = 11.68 \text{kJ/kg.K}$

State 2: $u_2 = h_2 - RT_2 = (-3770) - (0.5182)(350 + 273.15) = -4093 \text{kJ/kg}$

$\nu_2 = R \frac{T_2}{P_2} = (0.5182 \text{kJ/kg.K})(\frac{350 + 273.15 \text{K}}{4000 \text{kPa}}) = 0.08073 \text{m}^3/\text{kg}$

$w_{\text{ideal}} = P\nu_2 - u_1 = (4000 \text{kPa})(0.08073 - 0.04834)\text{m}^3/\text{kg} = 129.56 \text{kJ/kg}$

$q_{\text{ideal}} = w_{\text{ideal}} + (u_2 - u_1) = 129.56 + [(-4093) - (-4685)] = 721.70 \text{kJ/kg}$

$\Delta s_{\text{ideal}} = s_2 - s_1 = 11.68 - 10.22 = 1.46 \text{kJ/kg}$

For the generalized chart solution we first determine the following factors using EES as

\[
\begin{align*}
T_{R1} &= \frac{T_1}{T_{cr}} = \frac{373}{304.2} = 1.227 \\
p_{R1} &= \frac{P_1}{P_{cr}} = \frac{4}{7.39} = 0.5413 \\
T_{R2} &= \frac{T_2}{T_{cr}} = \frac{623}{304.2} = 2.048 \\
p_{R2} &= \frac{P_2}{P_{cr}} = \frac{4}{7.39} = 0.5413
\end{align*}
\]

\[\begin{align*}
Z_1 &= 0.9023, \quad Z_{\text{hi}} = 0.4318 \quad \text{and} \quad Z_{s1} = 0.2555 \\
Z_2 &= 0.995, \quad Z_{h2} = 0.1435 \quad \text{and} \quad Z_{s2} = 0.06446
\end{align*}\]

State 1:

$\Delta h_1 = Z_{h1}RT_{cr} = (0.4318)(0.5182 \text{kJ/kg.K})(304.2 \text{K}) = 68.07 \text{kJ/kg}$

$h_1 = h_{1,\text{ideal}} - \Delta h_1 = (-4492) - 68.07 = -4560 \text{kJ/kg}$

$u_1 = h_1 - Z_1RT_1 = (-4560) - (0.9023)(0.5182)(373.15) = -4734 \text{kJ/kg}$

$\nu_1 = Z_1R \frac{T_1}{P_1} = (0.9023)(0.5182)(\frac{373.15}{4000}) = 0.04362 \text{m}^3/\text{kg}$

$\Delta s_1 = Z_{s1}R = (0.2555)(0.5182 \text{kJ/kg.K}) = 0.1324 \text{kJ/kg.K}$

$s_1 = s_{1,\text{ideal}} - \Delta s_1 = 10.22 - 0.1324 = 10.09 \text{kJ/kg.K}$

State 2:

$\Delta h_2 = Z_{h2}RT_{cr} = (0.1435)(0.5182 \text{kJ/kg.K})(304.2 \text{K}) = 22.62 \text{kJ/kg}$

$h_2 = h_{2,\text{ideal}} - \Delta h_2 = (-3770) - 22.62 = -3793 \text{kJ/kg}$

$u_2 = h_2 - Z_2RT_2 = (-3793) - (0.995)(0.5182)(623.15) = -4114 \text{kJ/kg}$

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\[ \nu_2 = Z_2 R \frac{T_2}{P_2} = (0.995)(0.5182) \frac{623.15}{4000} = 0.08033 \text{ m}^3/\text{kg} \]

\[ \Delta s_2 = Z_2 s R = (0.06446)(0.5182 \text{ kJ/kg.K}) = 0.03341 \text{ kJ/kg.K} \]

\[ s_2 = s_{2,\text{ideal}} - \Delta s_2 = 11.68 - 0.03341 = 11.65 \text{ kJ/kg.K} \]

Then,

\[ w_{\text{chart}} = P(\nu_2 - \nu_1) = (4000 \text{ kPa})(0.08033 - 0.04362)\text{ m}^3/\text{kg} = 146.84 \text{ kJ/kg} \]

\[ q_{\text{chart}} = w_{\text{chart}} + (u_2 - u_1) = 146.84 + \left[ (-4114) - (-4734) \right] = 766.84 \text{ kJ/kg} \]

\[ \Delta s_{\text{chart}} = s_2 - s_1 = 11.65 - 10.09 = 1.56 \text{ kJ/kg} \]

The solution using EES built-in property data is as follows:

\[ T_1 = 100^\circ \text{C} \quad \nu_1 = 0.04717 \text{ m}^3/\text{kg} \]
\[ P_1 = 4 \text{ MPa} \quad u_1 = -39.82 \text{ kJ/kg} \]
\[ s_1 = -1.439 \text{ kJ/kg.K} \]

\[ T_2 = 350^\circ \text{C} \quad \nu_2 = 0.08141 \text{ m}^3/\text{kg} \]
\[ P_2 = 4 \text{ MPa} \quad u_2 = 564.52 \text{ kJ/kg} \]
\[ s_2 = 0.06329 \text{ kJ/kg.K} \]

\[ w_{\text{EES}} = P(\nu_2 - \nu_1) = (4000 \text{ kPa})(0.08141 - 0.04717)\text{ m}^3/\text{kg} = 136.96 \text{ kJ/kg} \]

\[ q_{\text{EES}} = w_{\text{EES}} + (u_2 - u_1) = 136.97 + \left[ 564.52 - (-39.82) \right] = 741.31 \text{ kJ/kg} \]

\[ \Delta s_{\text{EES}} = s_2 - s_1 = 0.06329 - (-1.439) = 1.50 \text{ kJ/kg} \]
**Fundamentals of Engineering (FE) Exam Problems**

**12-91** A substance whose Joule-Thomson coefficient is negative is throttled to a lower pressure. During this process, (select the correct statement)
(a) the temperature of the substance will increase.
(b) the temperature of the substance will decrease.
(c) the entropy of the substance will remain constant.
(d) the entropy of the substance will decrease.
(e) the enthalpy of the substance will decrease.

*Answer* (a) the temperature of the substance will increase.

**12-92** Consider the liquid-vapor saturation curve of a pure substance on the \( P-T \) diagram. The magnitude of the slope of the tangent line to this curve at a temperature \( T \) (in Kelvin) is (a) proportional to the enthalpy of vaporization \( h_{fg} \) at that temperature,
(b) proportional to the temperature \( T \),
(c) proportional to the square of the temperature \( T \),
(d) proportional to the volume change \( \nu_{fg} \) at that temperature,
(e) inversely proportional to the entropy change \( s_{fg} \) at that temperature,

*Answer* (a) proportional to the enthalpy of vaporization \( h_{fg} \) at that temperature,

**12-93** Based on the generalized charts, the error involved in the enthalpy of \( \text{CO}_2 \) at 350 K and 8 MPa if it is assumed to be an ideal gas is
(a) 0  
(b) 20%  
(c) 35%  
(d) 26%  
(e) 65%

*Answer* (d) 26%

*Solutions* Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[
T=350 \ "K" \\
P=8000 \ "kPa" \\
Pc=\text{P\_CRIT(CarbonDioxide)} \\
Tc=\text{T\_CRIT(CarbonDioxide)} \\
Tr=T/Tc \\
Pr=P/Pc \\
Z=\text{COMPRESS(Tr, Pr)} \\
hR=\text{ENTHDEP(Tr, Pr)}
\]
Based on data from the refrigerant-134a tables, the Joule-Thompson coefficient of refrigerant-134a at 0.8 MPa and 100°C is approximately

(a) 0  (b) -5°C/MPa  (c) 11°C/MPa  (d) 8°C/MPa  (e) 26°C/MPa

Answer (c) 11°C/MPa

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[ T1=100 \ ^\circ\text{C} \]
\[ P1=800 \ ^{k}\text{Pa} \]
\[ h1=\text{ENTHALPY(R134a,T=T1,P=P1)} \]
\[ T\text{low}=\text{TEMPERATURE(R134a,h=h1,P=P1+100)} \]
\[ T\text{high}=\text{TEMPERATURE(R134a,h=h1,P=P1-100)} \]
\[ JT=(T\text{low}-T\text{high})/200 \]

For a gas whose equation of state is \( P(\nu - b) = RT \), the specific heat difference \( c_p - c_v \) is equal to

(a) \( R \)  (b) \( R - b \)  (c) \( R + b \)  (d) 0  (e) \( R(1 + \nu b) \)

Answer (a) \( R \)

Solution The general relation for the specific heat difference \( c_p - c_v \) is

\[ c_p - c_v = -T\left(\frac{\partial \nu}{\partial T}\right)_P \left(\frac{\partial P}{\partial \nu}\right)_T \]

For the given gas, \( P(\nu - b) = RT \). Then,

\[ \nu = \frac{RT}{P} + b \] \[ \frac{\partial \nu}{\partial T} = \frac{R}{P} \]
\[ P = \frac{RT}{\nu - b} \] \[ \left(\frac{\partial P}{\partial \nu}\right)_T = -\frac{RT}{(\nu - b)^2} = -\frac{P}{\nu - b} \]

Substituting,

\[ c_p - c_v = -T\left(\frac{R^2}{P}\right)\left(-\frac{P}{\nu - b}\right) = \frac{TR^2}{P(\nu - b)} = R \]